# States' support of higher education: A theoretical and empirical analysis 

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# States' support of higher education: A theoretical and empirical analysis 

Holtkamp, Janice McClung, Ph.D.
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# States' support of higher education: A theoretical and empirical analysis 

by

Janice McClung Holtkamp

A Dissertation Submitted to the Graduate Faculty in Partial Fulfillment of the Requirements for the Degree of DOCTOR OF PHILOSOPHY

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## CHAPTER 1. INTRODUCTION

## Purpose

This dissertation uses public finance theory and econometrics to examine the variation of expenditures on public higher education among states. The basic theory of public goods and externalities combined with comparative static analysis provides the theoretical framework to examine the effect of possible explanatory variables on optimal subsidy levels for public higher education. Two theoretical models are developed, a one sector model of the higher education market that treats private education as independent of the public market, and a two sector model that recognizes the interdependence of the public and private markets for higher education. Crosssectional regression techniques are then used to determine the relative importance of differences in the explanatory variables on the optimal subsidy levels and to test the usefulness of separating the two higher education markets. One of the most significant findings is that states react to the vested interest they have in educating their citizens. Variables that influence future state government income are consistently significant in determining the optimal per-student public subsidy. This supports the argument that governments' behavior is affected by their equity interest in the educational investment of their citizens. Another significant finding is that state support of private higher education appears to decrease allocations to public higher education.

## Dissertation Organization

Chapter 2 reviews previous empirical studies on state expenditures for higher education. These studies provide a starting point for theoretical analysis of state support of public higher education as developed in Chapter 3.

Chapter 3 describes the theoretical foundation for a one-sector treatment of state support of public higher education. This chapter treats private higher education as essentially exogenous to the public higher education market in a state and uses comparative static analysis to provide insight
about the effects of changes or differences in the explanatory variables on the public higher education subsidy.

Chapter 4 describes the data used for the empirical models.
Chapter 5 outlines the empirical models implied by the theory in Chapter 3. These models are tested using linear regression techniques and conclusions are drawn based on the regression results.

Chapter 6 builds upon the higher education model in Chapter 3 by including the private higher education market in a two sector model of the higher education market in a state. In this section, private higher education is treated as endogenous to the model. Comparative static analysis is again employed to determine the effect of changes or differences in the explanatory variables on the optimal state subsidy for public higher education within the two-sector framework.

Chapter 7 includes the empirical models implied by the two sector theory developed in Chapter 6 and the conclusions that can be drawn from the results of the regression procedure.

Chapter 8 presents general conclusions from the previous chapters. The References, Acknowledgements and Appendices follow.

## CHAPTER 2. LITERATURE REVIEW

"The expense of the institutions for education and religious instruction, is likewise, no doubt, beneficial to the whole society, and may, therefore, without injustice, be defrayed by the general contribution of the whole society." (Adam Smith)
"All that is spent during many years in opening the means of higher education to the masses would be well paid for if it called out one more Newton or Darwin, Shakespeare or Beethoven." (Alfred Marshall)

It has long been recognized that investment in higher education by society is a necessary and beneficial endeavor. The rationale for government involvement is based on three arguments. One, there are imperfections in the market for higher education, specifically in the acquisition of financing for students. Since indentured servitude is illegal, students have difficulty securing loans for human capital investment because the lender has no collateral in case of default. The second argument for government involvement in the higher education market is that there are imperfections in the flow of information to students and their families. The result of this imperfection is that students simply lack the foresight to make efficient decisions about the long-term payoff versus short-run costs of higher education. The third justification is that there exist external benefits that accrue to society from the education of its members and these render the level of private investment inadequate (Wiseman, in Psacharopoulos, 1987). Subsidies, by all levels of government, have therefore been implemented in order to capture these benefits. How these subsidy amounts are determined is a question that has inspired three decades of research. In particular, there have been several studies of the variation among states' higher education appropriations, based on the research of Soloman Fabricant.

Fabricant, in his 1952 research, attempted to explain using regression analysis the primary determinants of per-capita state and local general expenditures. He found that per capita income, percent urban population and population density explained 72 percent of the variation in general state expenditures for 1942. Fabricant also examined specific expenditure categories such as local schools and highways but did not address higher education specifically. This preliminary study led
the way to more detailed analysis which included higher education expenditures, beginning with the work of Glenn Fisher.

Fisher, in his 1961 study, used multiple regression analysis to identify the "more important quantifiable factors and to present the 'unexplained' variations in such a form as to invite further analysis of both quantifiable and non-quantifiable influences" (Fisher, 1961). He used the explanatory variables of Fabricant (income, degree of urbanization and population density) to analyze state and local government expenditure data provided by the 1957 Census of Governments. In his study he estimated equations to determine expected expenditures for states and then compared the estimated and actual expenditures to discover the extent to which omitted variables influence expenditures.

Fisher found a coefficient of multiple correlation of .61 for his regression equation for higher education expenditure. The estimating equation showed that the only positive influence on higher education expenditures was per capita income.

In a 1964 study, Fisher attempted to explain a greater percentage of the variation of state expenditures and to indicate the relative importance of the variables studied. He again used regression analysis, and to demonstrate relative importance, computed multiple partial coefficients.

Fisher's original list of 12 explanatory variables was grouped into three categories: economic, demographic, and socio-political.

Fisher computed the multiple correlation coefficients with all 12 independent variables and numerous combinations until he arrived at the combination that resulted in the highest coefficient with the least number of variables. The seven included in the detailed analysis were the following.

1. Percent of families with incomes less than $\$ 2000$ in 1959 (X1),
2. yield of representative tax system, 1960, as percent of U.S. average (X2),
3. population per square mile in 1960 (X3),
4. percent of population in urban areas in 1960 (X4),
5. percent increase in population from 1950 to 1960 (X5),
6. Fenton's Index of two party competition (X6), and
7. percent of population over 25 with less than 5 years of schooling in 1960 (X7).

Fenton's Index was devised to test the hypothesis that two party competitiveness leads to government action on behalf of the less well-off in a state.

The seven variables incorporated in this analysis had a multiple coefficient of determination, $\mathbf{R}^{\mathbf{2}}$, of .59 for the regression equation for state and local expenditures on higher education. This was an improvement over the $\mathbf{R}^{\mathbf{2}}$ of .42 for the original three variables in the 1961 inquiry. Population per square mile, percent urban population, and percent of adults with less than five years schooling all were found to negatively influence higher education expenditures. Expenditures were positively influenced by percent of families with less than $\$ 2000$ incomes, tax yield, increase in population, and two party competition.

At the same time Fisher was adding to his original analysis of higher education expenditures, Seymour Sacks and Robert Harris were testing the importance of federal and state aid in explaining the variation of per capita spending between states. They did not address the issue of higher education specifically but rather included it in a "catch-all" category.

Sacks and Harris noted that the coefficients of multiple determination, $\mathbf{R}^{\mathbf{2}}$, in Fisher's two analyses had declined in many of the individual regression equations when compared to Fabricant's study. They felt this was due to the increasing importance of intergovernmental flows of funds and proceeded to test this hypothesis by using stepwise regression techniques.

The inclusion of federal aid to states as an independent variable was justified by Sacks and Harris because "federal aid can be regarded as 'outside money' from the point of view of the state and local government, and its availability should be expected to have a direct impact on raising state and local expenditure levels" (Sacks and Harris, p. 79). As Fisher notes though, results of studies that include federal aid must be interpreted with care because in the most extreme example, federal
matching programs will have a correlation of 1.0 with state expenditures and therefore cannot be considered as truly independent variables.

State aid was included as an independent variable to test the idea that it would lead to an increase in state and local expenditures combined. The issue of whether state and federal aid are in a sense the same variable was dismissed under the hypothesis that federal aid passed on from states to local governments would have a distinct expenditure effect compared to state aid to local governments and federal aid to state governments. Higher education was included in the "Not Specifically Aided and All Other" category, therefore no conclusions could be made about the effect of the two aid variables on expenditures except that they should be considered in future analyses.

Roy Bahl, Jr. and Robert Saunders included federal aid in their study of determining what factors influence changes in state and local government expenditures as opposed to what influences the magnitude of expenditures. Bahl and Saunders used multiple correlation analysis to discover how influential are changes in the independent variables when explaining changes in per capita general expenditures and per capita expenditures by function. The five explanatory variables used were:

1. change in per capita income,
2. change in population density,
3. change in urban population,
4. change in federal grants, and
5. change in public school enrollment.

The first three variables were employed by Fabricant in his 1952 study. The fourth was considered relevant due to the Sacks and Harris study. The final variable, public school enrollment, was included as a measure of the relative importance of educational expenditures in the total state and local budget.

The five variables explained 56 percent of the variation in higher education expenditures among the 48 states, with changes in urban population accounting for more than half of the variation. This result was even more pronounced when considering 15 high income, high density states alone. In that case, changes in urban population explained 79 percent of the variation in higher education expenditures. This is significant since the five variables together explain 89 percent of the variation in higher education expenditures.

In 1969 M. Charles McIntyre designed a model to look at interstate variations in public higher education expenditures; it included variables external to the institution as well as internal variables. McIntyre especially wanted to examine the effect of funding source differences among three types of public institutions: universities, four-year colleges, and two-year junior colleges.

McIntyre employed the following model and ran regressions based on it for the alternative institution types.
$\mathbf{E}=\mathbf{f}(\mathbf{S}, \mathbf{A}, \mathbf{T}, \mathbf{I})$ where;
$E=\quad$ reported expenditure levels per student for instruction,
$S=\quad$ description of student enrollment by class level and extent of full-time attendance,
$A=\quad$ general financial ability of the state to support public higher education,
$T=\quad$ index measuring tax effort for public higher education, and
$I=\quad$ relative utilization of different types of income sources.

The student enrollment variable, S, was included on the hypothesis that institutions that enroll primarily graduate students would be expected to have higher expenditure per student due to smaller class size, fewer teaching units per faculty, and possibly higher average faculty salaries. The variable $S$ also incorporates the extent of full-time versus part-time attendance because it is probable that costs differ between the two. The first partial derivative of $\mathbf{E}$ with respect to $S$ is expected to be positive.

Variable A, the ability of the state to finance public higher education, is in terms of personal income per capita but McIntyre acknowledges that a more relevant measure would look at such factors as age distribution, natural resources, geographical location and distribution of the population between urban and rural areas. This is very difficult to apply in practice so he deferred to the traditional measure. The first partial derivative of $E$ with respect to $A$ is also expected to be positive since it is likely that wealthier states, ceteris paribus, would spend more for higher education than less wealthy states.

Tax effort, $T$, is the amount of tax revenue raised for higher education relative to total personal income. This is corrected for population differences to give an idea of the relative value of the resources given up to the public sector by a citizen. The effect of tax effort on the expenditure function is expected to be positive.

The variable I describes the possible income sources and their relative importance to the expenditure function. This variable encompasses tuition, fees, state aid and local government aid. The sign of the derivative of expenditures with respect to income sources is ambiguous since reliance on a particular type of income is not indicative of higher or lower expenditures per student by institutions.

McIntyre conducted an empirical test and found that his model explained 50 percent of the variation between states in instructional expenditures and almost 80 percent of the variation in expenditures in four-year public colleges.

McIntyre found that in the university expenditure regression, all but two variables were significant at the one percent level. Type of degree granted and tax effort appeared to have little impact on university instructional spending per student. The results for the different types of income showed that reliance on one type of income source did not seem to affect variation in expenditures when adjusted for student characteristics, state financial ability and tax effort. Interestingly,

McIntyre determined from correlation coefficients that increases in tuition did not act as a substitute for state tax support; rather, they were more than matched by increases in state aid.

Further advances in explaining higher education expenditure variations were made by Robert Peterson. His study of the determinants of state higher education appropriations in 1976 included 20 socioeconomic, institutional environment, and political variables, many not used in previous analyses.

Peterson divided the dependent variables, per-capita and per-student appropriations for 1960 and 1969, into three categories based on institutional type: four-year and above public institutions, two-year public institutions and all public institutions of higher education. The socioeconomic independent variables considered were the following.

1. Hofferbert's industrialization factor scores,
2. Hofferbert's affluence factor scores,
3. personal income per capita,
4. corporate income per capita,
5. median years of school completed by population ( 25 years or older),
6. percent of population 25 years and older that is college educated,
7. Percent of population of college age (18 to 22 ).

The two factor scores of Hofferbert are indexes that are derived from a number of weighted variables. The industrialization score is influenced by such things as manufacturing employment, population density, urban population, value of farm property and other less apparent variables such as telephones. The affluence factor score is comprised of such variables as median school years, real property values, personal income, illiteracy, telephones and motor vehicles. This factor is strongly affected by wealth and educational attainment of the population and is expected to influence a state's demand for education.

The two income measures and the two education measures are expected to positively affect appropriations because it is likely that wealthier, educated populations have a stronger motivation to support higher education.

The percent of college-age population, preferences for private versus public schooling constant, is also expected to positively influence the demand for higher education.

The institutional environment variables were the following.
8. All public institution enrollment per 10,000 population,
9. public junior college enrollment per 10,000 population,
10. public senior institution enrollment per 10,000 population,
11. all private higher education enrollment per $\mathbf{1 0 , 0 0 0}$ population,
12. number of private institutions per one million population,
13. public institution students (in state or out of state) as a percent of all college student state residents, and
14. advanced degree work as a percent of public enrollment.

The enrollment measures are included to determine their impact upon policy decisions. Number of private institutions is included based on the idea that they have the potential to pressure policy makers for their own needs. The percent of all college students attending public institutions is included to reflect the public - private bias of the state, with the idea that a state more oriented towards private higher education is less apt to devote many resources to public higher education. Finally, advanced degree work is included, as in the McIntyre study, to determine whether this results in higher appropriations due to the greater expense entailed.

The six political measures incorporated in the study are the following.
15. Sharkansky and Hofferbert's professionalism-local reliance factor scores,
16. Sharkansky and Hofferbert's competition-turnout factor scores,
17. Schlesinger's combined index of governor's powers,
18. Francis' index of centralization in decision-making,
19. McCrone and Cnudde's anti-discrimination index,
20. Walker's innovation scores.

The Sharkansky and Hofferbert professionalism-local reliance factor was designed to indicate state legislative professionalism as well as reliance on state and federal funds versus local funds. It is hypothesized that professional legislators would likely develop the highest level of public services attainable as opposed to amateur legislators. The competition-turnout factor was developed to measure interparty competition and voter turnout. States with high competition-turnout factor scores are expected to have higher public spending.

The index of governor's powers was included on the premise that a governor with strong powers would likely favor higher education. Francis' index of centralized decision making is incorporated to test political leadership roles in general. The anti-discrimination index measures the support of civil rights in a state and may influence appropriations positively or negatively depending on civil rights attitudes and the extent that legislators perceive colleges to be influential in this area. Lastly, the innovation score was included with the idea that if public higher education institutions are seen as contributors to innovative ideas, they may gain or lose depending upon a state's tendency towards innovation.

Peterson used correlation coefficients and regression analysis to measure the significance and importance of the preceding 20 variables in determining state appropriations for higher education. The leading socioeconomic variable affecting per capita appropriations was educational attainment of the population. The affluence factor yielded almost identical results, which is understandable since educational attainment is the most highly weighted component of the factor. An interesting result for the senior institution sector was that median school years was significant while the percent college educated was not, and both were significant for the junior college sector. Peterson suggests that "this may reflect a higher level of confidence by college educated parents that their childrens'
needs will be fulfilled by existing four year college opportunities and a preference for the junior college option for 'other' peoples children" (Peterson, p. 531).

The only significant negative relationship found was between per-capita appropriations and the industrialization index. Peterson notes that this may be due to the strong relationship between private college orientation and industry and that the negativity is really due to the influence of private college orientation.

Personal income and percent of college-age population did not emerge as significant determinants of per-capita appropriations for either 1960 or 1969 but personal income did influence per-student expenditures for senior institutions.

Median years of schooling and affluence affected per-student expenditures positively but were weaker in their influence on per-capita spending in 1960. In 1969 they were insignificant. Percent of the population college-aged was significant in both 1960 and 1969 for per-student spending but not for per-capita results. The author determined that the increased influence of college-aged population supports the hypothesis that private school orientation no longer negatively affects financial support of students enrolled in public universities.

The public enrollment variables exhibit the strongest effects on per-capita appropriations for the relevant institution types. As would be expected, enrollment in senior institutions has the greatest effect on appropriations for senior institutions per capita for 1960 and 1969. This relationship, positive and quite strong, $k$ Ids for all of the enrollment variables and institution types.

The private enrollment and private college variables are also significant for per-capita spending for the two years examined and, not surprisingly, are negative. This negativity declined between 1960 and 1969 and is likely due to the growth in the demand for higher education and increased pressure for greater accessibility in the 1960s.

When comparing per-student spending to per-capita spending, the results are decidedly different. The public enrollment variables almost all become insignificant and those that are
significant change signs and negatively influence expenditures. Peterson notes that the end of the 60s saw states with large public sectors declining relatively in subsidization, having possibly reached their upper limits.

As in McIntyre's analysis, percent of advanced degree work was insignificant in both per-capita and per-student models. The author suggests that the greater cost of graduate education must come at the expense of other education levels.

The political variables included in the study were found to have some influence on state spending. The competition-turnout variable was significant for per capita appropriations for all public and senior public institutions. This supports the hypothesis that interparty competition leads to greater expenditure levels and is in agreement with Fisher's 1964 results.

The professionalism-local reliance results are mixed but the author states that since it is significantly positive in 1969 for per-student spending in senior institutions, this implies that a professional legislature is likely to support high per-student subsidization.

Peterson's study extended previous analyses of higher education appropriations and yielded many interesting results. The most significant finding according to him was that the institutional environment variables were the most important in explaining state spending. That is, in states with higher enrollment per capita, there were significantly higher appropriations.

The aforementioned five studies have made great advances in determining what influences government support of higher education and the level of expenditures allocated for instructional purposes. The following chapters of this dissertation build upon these previous studies by providing a theoretical basis for comparing interstate variation in support of higher education. The theory of externalities is employed to justify the inclusion of the variables used in the empirical models, some of which were included in earlier works. The theory also identifies some variables not considered in previous studies.

## CHAPTER 3. ONE-SECTOR MODEL OF HIGHER EDUCATION

## A Theoretical Model

The theory of externalities can be used to depict the nature of public higher education.

## Consider the diagram in Figure 1.

The private demand curve, $D_{E}$, is the horizontal summation of the individuals' demands for public higher education. This is continuous in the aggregate even though at any given price an individual will make an all-or-nothing decision about investing in education. $D_{E}$ reflects the amount of public education privately demanded in a state by its' citizens at various tuition prices.

The pseudo public demand curve for public higher education, PVEB (Present Value of External Benefits), is the public valuation of the present and future external benefits from education. PVEB is the vertical summation of the representative individual's valuation of the external benefits, IVEB. Apparently, PVEB is located everywhere above the private demand curve $\mathrm{D}_{\mathrm{E}}$ because at any enrollment level, tuition is typically less than the expenditure by government. In 1987-1988, tuition was 18.8 percent of the $\$ 47$ billion four-year public institution revenue while state and local government contributions constituted 53 percent (State Higher Education Profiles).


Figure 1: Market Equilibrium with Externalities

The two demand curves, $\mathrm{D}_{\mathrm{E}}$ and PVEB, can be summed vertically to arrive at the total demand curve for public higher education in a state, $D_{s}$. This demand curve interacts with the supply curve for public higher education, $S_{E}$, to determine the optimal tuition, subsidy and quantity, $P_{E}{ }^{\circ}, S^{\circ}$, and $\mathrm{Q}_{\mathrm{E}}{ }^{\text {a }}$ respectively. Clearly, the private amount of public higher education that would be purchased, Q , is suboptimal if there are marginally relevant positive externalities; these give rise to the need for some type of subsidization. There are several measures of subsidies that can be calculated, as portrayed by the diagram in Figure 2.


Figure 2: Alternative Subsidy Measures for Public Higher Education

The subsidy needed to achieve the optimal enrollment level can be determined by the height of the public demand schedule PVEB at $Q_{E}{ }^{\circ}$. Alternatively, it is the vertical distance between $D_{S}$ and $\mathrm{D}_{\mathrm{E}}$ at $\mathrm{Q}_{\mathrm{E}}{ }^{*}$. This subsidy corresponds to the annual per-student subsidy, $\mathrm{S}_{3}$, allocated by the state. Multiplying $S_{3}$ by enrollment gives the total amount of a state's annual appropriations to public
higher education, $S_{4}$. On a smaller scale, the height of IVEB at $Q_{E}{ }^{\bullet}$ reflects the amount that the representative citizen is willing to contribute per student, $S_{1}$, to enjoy the external benefits. Multiplying by total enrollment gives the annual per-citizen subsidy to public higher education, $\mathbf{S}_{\mathbf{2}}$.

Consider the following example of 1988 expenditures for Iowa. The Iowa legislature appropriated $\$ 369,579,126$ for public university support. This figure can be considered as the equivalent of $S_{4}$ if appropriations are based on this theoretical analysis. Enrollment in the three universities was 59,030 full-time-equivalent students. The annual per-student subsidy, $S_{3}$, would then amount to $\$ 6,260.86$. Iowa's population in 1988 was $2,834,000$. The per-capita subsidy to public higher education, $S_{2}$, was $\$ 130.40$ and the per-capita, per-student subsidy amount, $S_{1}$, was $\$ 0.0022$ (Statistical Abstract of the United States; State Higher Education Profiles; IPEDS).

The level of these subsidies depends upon the location of the supply and demand schedules for public higher education in each state and the valuation of the external benefits. The external benefits curve however, is a function of total enrollment and is therefore dependent upon both private and public higher education enrollments. In this section of the theory, the private higher education market will be considered as exogenous. The relationship between the two markets will be explored in greater detail in Chapter 6.

The determinants of the schedules in the preceding figures have been examined in previous works and will be discussed in the analysis, with the emphasis on those variables expected to vary across states. The following have been identified as potentially influential.

1. Percent of population college-aged.
2. Average educational attainment.
3. Per-capita income.
4. Population.
5. Percent urban population.
6. Population density.
7. Expected increase in real after-tax future earnings resulting from a college education.
8. Foregone earnings while attending college.
9. Marginal tax rate.
10. Migration.
11. Grants, contributions, endowment income, federal aid.
12. Percent junior college enrollment.
13. Private school orientation.
14. State aid to private schools.

Many of the explanatory variables affect more than one curve. Explanations of how and why these affect the respective curves will be discussed in greater detail in subsequent pages. The private demand curve for public education, $\mathrm{D}_{\mathrm{E}}$, is considered here to be a function of the following independent variables: percent of the population college-aged, average educational attainment, percapita income, population, population density, expected increase in real after-tax future earnings, foregone earnings, marginal tax rate, private school orientation, and state aid to private institutions.

PVEB, the present value of external benefits curve, is influenced by the following variables: average educational attainment, per-capita income, population, percent urban population, population density, expected increase in real after-tax future earnings, foregone earnings, marginal tax rate, migration, private school orientation and state aid to private institutions.

The supply curve for public education, $\mathrm{S}_{\mathrm{E}}$, is affected by the following: per-capita income, percent urban population, grants, contributions, endowment income and federal aid, and junior college enrollment.

Figure 3 and equations 1-6 describe an equilibrium.

$$
\begin{array}{ll}
\text { PVEB: } S=A-a Q_{E} & \text { with } A=\sum_{i=1}^{n} \alpha_{1} X_{i} \\
D_{E}: P_{E}^{D}=B-b Q_{E} & \text { with } B=\sum_{i=1}^{n} \gamma_{i} X_{i} \tag{2}
\end{array}
$$

$$
\begin{equation*}
S_{\dot{\Sigma}} P_{E}^{s}=C+c Q_{E} \quad \text { with } C=\sum_{i=1}^{n} \theta_{i} X_{i} \tag{3}
\end{equation*}
$$

where $A, B, C, a, b, c, \alpha^{i}, \gamma_{i}$, and $\theta_{i}$ are parameters, $Q_{B}$ is enrollment, $P_{E}$ is tuition, $S$ is the perstudent subsidy and the $\mathrm{X}_{\mathrm{i}} \mathrm{s}$ are the explanatory variables. The intercepts, $\mathrm{A}, \mathrm{B}$, and C depend on several factors which are expected to vary across states. Differences or changes in these factors will be represented by shifts in the intercepts of the schedules. The equilibrium condition is: $\mathrm{P}_{\mathrm{E}}{ }^{\mathrm{D}}+\mathrm{S}=$ $\mathbf{P}_{\mathrm{E}}{ }^{\mathbf{s}}$.


Figure 3: Equilibrium in the Public Higher Education Market
The optimal solutions are:
(4)

$$
S^{*}=A-\frac{a(A+B-C)}{a+b+c}
$$

(5)

$$
P_{E}^{\cdot}=B-\frac{b(A+B-C)}{a+b+c}
$$

$$
\begin{equation*}
Q_{\dot{E}}^{\bullet}=\frac{A+B-C}{a+b+c} . \tag{6}
\end{equation*}
$$

$S^{\prime}$ is the optimal per-student subsidy and $P_{\mathrm{E}}{ }^{\prime}$ is the optimal tuition price to charge to students when enrollment is at $\mathrm{Q}_{\mathrm{E}}{ }^{\circ}$.

Comparative statics can be used to analyze the effect of a change or differences in one of the explanatory variables on the optimal subsidy level. The effect of a change is an indirect one. The explanatory variables influence the location of the demand, supply and external benefits schedules resulting in the optimal subsidy level and the other variables in the model adjusting to a changed equilibrium.

## Comparative Static Analysis

## Percent of the Population College-aged

Consider the effects of a change in the percent of the population college-aged on the public higher education market. An increase in this percentage is shown here affecting only the private demand curve for public education. A state with a large proportion of college-aged citizens will have a greater private demand for public higher education, ceteris paribus. This effect is illustrated by an upward or rightward shift in the private demand curve for public education from $D_{E}$ to $D_{E I}$, in Figure 4. As discussed earlier, when there is a change in $\mathrm{D}_{\mathrm{E}}$ or in the present value of external benefits curve, PVEB, the total demand for public higher education, $\mathrm{D}_{\mathrm{s}}$, also changes. In this instance, the upward shift in $D_{E}$ to $D_{E 1}$ results in an upward shift in $D_{s}$ to $D_{S 1}$. The new equilibrium point at $E_{1}$ is characterized by a decrease in the per-student subsidy to $S_{1}$, an increase in tuition levels to $P_{E 1}$, and an increase in total enrollment to $Q_{E 1}$. The increase in tuition is not readily apparent but can be seen by comparing the vertical distance between $\mathrm{D}_{\mathrm{s} 1}$ and PVEB at $\mathrm{Q}_{\mathrm{E} 1}$ to the vertical distance between the original $D_{s}$ curve and PVEB at $\mathrm{Q}_{\mathrm{E}}{ }^{\circ}$.


Figure 4: The Effects of a Change in the Percent of the Population College-aged on the Public Higher Education Equilibrium

The effects of a change in the percentage of college-aged population shown above graphically can also be demonstrated mathematically. Using equations (1), (2), (3) and (4), the effect of a change in one of the independent variables on the optimal subsidy level can be seen. In general,

$$
\begin{equation*}
\frac{\partial S^{*}}{\partial X_{i}}=\frac{\partial S^{*}}{\partial A} \frac{\partial A}{\partial X_{i}}+\frac{\partial S^{*}}{\partial B} \frac{\partial B}{\partial X_{i}}+\frac{\partial S^{*}}{\partial C} \frac{\partial C}{\partial X_{i}} \tag{7}
\end{equation*}
$$

Using equation (4),

$$
\frac{\partial S^{*}}{\partial A}=\frac{b+c}{a+b+c} ; \frac{\partial S^{*}}{\partial B}=-\frac{a}{a+b+c} ; \frac{\partial S^{*}}{\partial C}=\frac{a}{a+b+c} .
$$

Substituting these into (7) gives

$$
\begin{equation*}
\frac{\partial S^{*}}{\partial X_{i}}=\frac{b+c}{a+b+c} \frac{a A}{\partial X_{i}}-\frac{a}{a+b+c} \frac{\partial B}{\partial X_{i}}+\frac{a}{a+b+c} \frac{\partial C}{\partial X_{i}} \tag{7.1}
\end{equation*}
$$

Let the percentage of college-aged citizens be denoted as $X_{1}$. Then, replacing $X_{i}$ with $X_{1}$ in equation (7.1) and substituting in the partials of the intercept terms with respect to $X_{1}$ gives

$$
\begin{equation*}
\frac{\partial S^{\bullet}}{\partial X_{1}}=-\frac{a}{a+b+c} \gamma_{1}<0 \tag{8}
\end{equation*}
$$

Since $a, b, c$ and $\gamma_{1}>0$, this partial derivative is clearly negative indicating that an increase in the percentage of college-aged citizens lowers the optimal per-student subsidy.

In a similar fashion, the effect of an increase in the percentage of college-aged population on tuition can be demonstrated. Using equations (1); (2), (3), and (5),

$$
\begin{equation*}
\frac{\partial P_{E}^{*}}{\partial X_{i}}=\frac{\partial P_{E}^{*}}{\partial A} \frac{\partial A}{\partial X_{i}}+\frac{\partial P_{E}^{*}}{\partial B} \frac{\partial B}{\partial X_{i}}+\frac{\partial P_{E}^{*}}{\partial C} \frac{\partial C}{\partial X_{i}} \tag{9}
\end{equation*}
$$

Using equation (5),

$$
\frac{\partial P_{E}^{*}}{\partial A}=-\frac{b}{a+b+c} ; \frac{\partial P_{E}^{*}}{\partial B}=\frac{a+c}{a+b+c} ; \frac{\partial P_{E}^{*}}{\partial C}=\frac{b}{a+b+c}
$$

Substituting these into (9) gives

$$
\begin{equation*}
\frac{\partial P_{E}^{*}}{\partial X_{i}}=-\frac{b}{a+b+c} \frac{\partial A}{\partial X_{i}}+\frac{a+c}{a+b+c} \frac{\partial B}{\partial X_{i}}+\frac{b}{a+b+c} \frac{\partial C}{\partial X_{i}} \tag{9.1}
\end{equation*}
$$

Replacing $X_{i}$ with $X_{1}$ in (9.1) and inserting the appropriate partials gives the effect of a change in
the percentage of the population college-aged on tuition levels.

$$
\begin{equation*}
\frac{\partial P_{E}^{*}}{\partial X_{1}}=\frac{a+c}{a+b+c} \gamma_{1}>0 \tag{10}
\end{equation*}
$$

Consistent with Figure 4, an increase in the percentage of the college-aged population would result in an increase in tuition levels.

The change in enrollment can be shown using equations (1), (2), (3), and (6),

$$
\begin{equation*}
\frac{\partial Q_{\dot{E}}^{*}}{\partial X_{i}}=\frac{\partial Q_{E}^{*}}{\partial A} \frac{\partial A}{\partial X_{i}}+\frac{\partial Q_{E}^{*}}{\partial B} \frac{\partial B}{\partial X_{i}}+\frac{\partial Q_{E}^{*}}{\partial C} \frac{\partial C}{\partial X_{i}} \tag{11}
\end{equation*}
$$

Using equation (6),

$$
\frac{\partial Q_{E}^{*}}{\partial A}=\frac{1}{a+b+c} ; \frac{\partial Q_{E}^{*}}{\partial B}=\frac{1}{a+b+c} ; \frac{\partial Q_{E}^{*}}{\partial C}=-\frac{1}{a+b+c}
$$

Substituting these into (11) gives

$$
\begin{equation*}
\frac{\partial Q_{E}^{*}}{\partial X_{i}}=\frac{1}{a+b+c} \frac{\partial A}{\partial X_{i}}+\frac{1}{a+b+c} \frac{\partial B}{\partial X_{i}}-\frac{1}{a+b+c} \frac{\partial C}{\partial X_{i}} . \tag{11.1}
\end{equation*}
$$

Replacing $X_{i}$ with $X_{1}$ in (11.1) and substituting in the relevant partials shows the effect of a change in the percentage of the college-aged population on enrollment levels.

$$
\begin{equation*}
\frac{\partial Q_{E}^{*}}{\partial X_{1}}=\frac{1}{a+b+c} \gamma_{1}>0 \tag{12}
\end{equation*}
$$

As with tuition, the sign of this partial is positive and therefore an increase in the percentage of the college-aged population would result in an increase in enrollment.

## Average Educational Attainment

Average educational attainment influences more than one curve. As families' average education level rises, they demand more education for their offspring. Borus and Carpenter found in their 1984 study of college attendance by high school seniors that seniors whose fathers have at least one year of college have probabilities of 19-26 percentage points higher of attending college than seniors who did not (Borus and Carpenter, 1984). Parsons work on education decisions by males discusses the "intergenerational effect" of education, that is, youth are more likely to complete a given level of education if their parents are more highly educated (Parsons, 1974).

An increase in average educational attainment would have the effect of shifting the private demand curve, $\mathrm{D}_{\mathrm{E}}$, upward to $\mathrm{D}_{\mathrm{E} 1}$ and therefore would shift the total demand curve for public higher education, $D_{s}$, upward to $D_{s 1}$ as shown in Figure 5 . If this were the only change, the new equilibrium, $E_{1}$, would have a higher enrollment of $Q_{E 1}$, a lower subsidy of $S_{1}$, and higher tuition of $P_{E 1}$. Again, the increase in tuition can be seen by comparing the vertical distance between $D_{S 1}$ and PVEB at $Q_{E I}$ to the vertical distance between $D_{s}$ and PVEB at $Q_{E}^{*}$.


Figure 5: The Effects of an Increase in Average Educational Attainment on Public Higher Education Equilibrium

At the same time that $D_{\mathrm{B}}$ shifts, the external benefits curve, PVEB, would also shift upward. As educational attainment rises, the individual's willingness to pay for educated neighbors also rises. This willingness to pay could be reffected in an upward shift of the IVEB curve, the individual valuation of the external benefits of education. Summing the IVEB curves vertically over the population gives the new external benefits curve $\mathrm{PVEB}_{1}$ which is located everywhere above the original curve. If the shift of PVEB occurred without the change of private demand, the new equilibrium point could again be illustrated by $\mathrm{E}_{1}$. This equilibrium point would be distinguished by enrollment rising to $Q_{E 1}$, tuition falling to $P_{E_{22}}$, and the optimal subsidy increasing to $S_{2}$. The subsidy change can be seen by comparing the vertical distance between $D_{S}$ and $D_{E}$ at $Q_{E}{ }^{\circ}$ to the vertical distance between $D_{S 1}$ and $D_{E}$ at $Q_{E 1}$.

The combined effect of both $\mathrm{D}_{\mathrm{E}}$ and PVEB responding to an increase in educational attainment would be to increase the $D_{s}$ schedule even more than shown in Figure 5. Thus, enrollment would rise but the total effect on the subsidy and tuition levels is indeterminant. This result can be confirmed mathematically. Let educational attainment be represented by $\mathrm{X}_{2}$. Using equation (7.1), substituting $\mathbf{X}_{\mathbf{2}}$ for $\mathbf{X}_{\mathrm{i}}$, and inserting the appropriate partials for the intercept terms with respect to average educational attainment, the effect of a change in educational attainment on subsidy level can be expressed as follows:

$$
\begin{equation*}
\frac{\partial S^{*}}{\partial X_{2}}=\frac{b+c}{a+b+c} \alpha_{2}-\frac{a}{a+b+c} \gamma_{2} \geq 0 . \tag{13}
\end{equation*}
$$

Because all of these parameters are positive, the sign of this expression is indeterminate. Thus, the effect of an increase in educational attainment on the optimal per-student subsidy is ambiguous.

Substituting $X_{2}$ for $X_{i}$ in equation (9.1) shows the change in tuition from a change in average educational attainment.

$$
\begin{equation*}
\frac{\partial P_{E}^{*}}{\partial X_{2}}=-\frac{b}{a+b+c} \alpha_{2}+\frac{a+c}{a+b+c} \gamma_{2} \geq 0 \tag{14}
\end{equation*}
$$

Therefore, an increase in educational attainment has an ambiguous effect on tuition.
The same procedure can be used to depict the change in enrollment. Replacing $\mathbf{X}_{\mathrm{i}}$ with $\mathbf{X}_{\mathbf{2}}$ in equation (11.1) and inserting the partials of the intercepts with respect to educational attainment gives

$$
\begin{equation*}
\frac{\partial Q_{E}^{*}}{\partial X_{2}}=\frac{\alpha_{2}+\gamma_{2}}{a+b+c}>0 \tag{15}
\end{equation*}
$$

As demonstrated by (15), enrollment rises when average educational attainment rises.

## Per-Capita Income

Consider the implications of a change in per-capita income on the public higher education market. As income rises, the private demand for public higher education will also rise since education is a normal good and parents are better able to finance the cost of their children's education. Radner and Miller found in their analysis of 1966 high school seniors that parental income positively affected the decision of whether to attend college (Radner and Miller, 1970). Others, including Campbell and Seigel (1967), and Corrazini, Dugan and Grabowski (1972) have also found income to be statistically significant in the demand for higher education. This means that $\mathbf{D}_{\mathbf{E}}$ will shift rightward, illustrating that at every tuition price, people are willing and able to purchase more education. Considered alone, the shift in $D_{E}$ would raise $D_{S}$ and thereby result in an increase in enrollment, a decrease in the amount of subsidy needed, and an increase in tuition levels.

The shift in the private demand curve for public education, however, is not the only curve affected by per-capita income. The present value of external benefits curve, PVEB, may also be affected with a rise in per-capita income. As income rises for a state's citizens, the amount that the median citizen is willing and able to pay to enjoy the benefits of his neighbors' education will also rise. As in the previous example of average educational attainment, this could be reflected by an upward shift in IVEB, the individual's valuation of the external benefits from education. The
location of the new PVEB curve can be found by summing the IVEB curves vertically across the population. The upward shift in PVEB would cause the location of $D_{s}$ to shift upward also. The new equilibrium point, if this were the only change, would be one in which enrollment and the optimal subsidy rise and tuition falls.

In addition to the shifts in $\mathrm{D}_{\mathrm{E}}$ and PVEB, a change in per-capita income would have the effect of shifting upward the supply curve for higher education, $\mathrm{S}_{\mathrm{E}}$. Cohn found in his 1973 study that institutions of higher education that are in high per-capita income states tend to pay higher faculty salaries. The upward shift in $\mathrm{S}_{\mathrm{z}}$ considered alone would result in an equilibrium characterized by lower enrollment, higher tuition and higher subsidy levels.

Let per-capita income be expressed as $X_{3}$. Then, replacing $X_{i}$ with $X_{3}$ in equation (7.1) and substituting in the appropriate partials for the intercept terms with respect to per-capita income gives

$$
\begin{equation*}
\frac{\partial S^{*}}{\partial X_{3}}=\frac{b+c}{a+b+c} \alpha_{3}-\frac{a}{a+b+c} \gamma_{3}+\frac{a}{a+b+c} \theta_{3} \geq 0 . \tag{16}
\end{equation*}
$$

With all the parameters in (16) greater than zero, the effect of a change in per-capita income on subsidy level is indeterminate and depends on the magnitude of the shifts in the three curves.

Tuition changes from an increase in per-capita income are also dependent upon the magnitude of the shifts in the three curves and can be shown to be ambiguous.

$$
\begin{equation*}
\frac{\partial P_{E}^{*}}{\partial X_{3}}=-\frac{b}{a+b+c} \alpha_{3}+\frac{a+c}{a+b+c} \gamma_{3}+\frac{b}{a+b+c} \theta_{3} \geq 0 \tag{17}
\end{equation*}
$$

The change in enrollment can be seen to be indeterminant using equation (11.1). Replacing $X_{i}$ with $X_{3}$ and inserting the suitable partials with respect to the intercepts gives

$$
\begin{equation*}
\frac{\partial Q_{E}^{*}}{\partial X_{3}}=\frac{\alpha_{3}+\gamma_{3}-\theta_{3}}{a+b+c} \geq 0 . \tag{18}
\end{equation*}
$$

## Population

Historically, population has been included in state appropriations analyses. Population must be included when considering total state appropriations to public higher education and per-student appropriations to account for differences due solely to population size. Ceteris paribus, an increase in the population of a state would shift the demand curve for public higher education upward to $\mathrm{D}_{\mathrm{EI}}$ from $D_{B}$, reflecting the increased number of consumers. Hence, the total demand curve for public higher education would also shift upward from $D_{s}$ to $D_{s 1}$. The new equilibrium, $E_{1}$, previously illustrated in Figure 5, would be characterized by a higher enrollment of $Q_{E 1}$, a lower subsidy of $S_{1}$, and a higher tuition level of $\mathbf{P}_{\mathrm{El}}$.

Population affects the external benefits curve, PVEB, for public higher education as well. Ceteris paribus, an increase in the population of a state would result in an upward shift of the PVEB curve since the PVEB curve is the vertical summation, over the state's population, of the individual valuations of the external benefits of education, IVEB. As demonstrated in Figure 5, an upward shift in PVEB to PVEB $_{1}$ would cause the total demand curve for public higher education to shift upward to $D_{S 1}$ from $D_{s}$. The effect of this would be a new equilibrium, $E_{1}$, with a higher enrollment level of $\mathrm{Q}_{\mathrm{E} 1}$, a lower tuition of $\mathrm{P}_{\mathrm{E} 2}$, and an increased subsidy level of $\mathrm{S}_{2}$.

The combined effect of an increase in a state's population would be to increase enrollments and would be indeterminate with respect to tuition and subsidy. Let population of a state be $\mathbf{X}_{4}$. Then, replacing $X_{i}$ with $X_{4}$ in equation 7.1 and substituting in the appropriate partials for the intercept terms with respect to population gives

$$
\begin{equation*}
\frac{\partial S^{*}}{\partial X_{4}}=\frac{b+c}{a+b+c} \alpha_{4}-\frac{a}{a+b+c} \gamma_{4} \geq 0 . \tag{19}
\end{equation*}
$$

Since $\alpha_{4}$ and $\gamma_{4}$ are positive, a change in the population of a state on subsidy level is indeterminate.

The effect of a change in a state's population on tuition can be seen to be ambiguous with equation (20).

$$
\begin{equation*}
\frac{\partial P_{E}^{*}}{\partial X_{4}}=-\frac{b}{a+b+c} \alpha_{4}+\frac{a+c}{a+b+c} \gamma_{4} \geq 0 \tag{20}
\end{equation*}
$$

The predicted increase in enrollment is demonstrated by equation (21).

$$
\begin{equation*}
\frac{\partial Q_{E}^{*}}{\partial X_{4}}=\frac{\alpha_{4}+\gamma_{4}}{a+b+c}>0 \tag{21}
\end{equation*}
$$

## Urbanization

The degree of urbanization has been included in public higher education expenditure studies since Fabricant's 1952 analysis. Urbanization is expected to influence the optimal subsidy for public higher education because as a state becomes increasingly congested and industrialized, the individual valuation of the external benefits of higher education, IVEB, is expected to rise. This is because citizens value having educated neighbors and the corresponding increase in economic development as well as lower crime rates, decreased welfare costs, increased community service, etc. associated with higher education. The upward shift in IVEB from higher education would result in an upward shift of the PVEB curve. As illustrated in Figure 6, the movement from PVEB to PVEB $_{1}$ causes an upward shift in the total demand curve for public higher education from $D_{s}$ to $D_{s 1}$ and would result in an equilibrium at $E_{1}$. The new equilibrium point would be characterized by higher enrollment, lower tuition and higher subsidy levels, $\mathrm{Q}_{\mathrm{El}}, \mathrm{P}_{\mathrm{EI}}$, and $\mathrm{S}_{1}$ respectively.

Increased urbanization may also affect the supply curve for public higher education, $\mathbf{S}_{\mathrm{E}}$, because higher wage rates are associated with greater urbanization. This would imply that $\mathrm{S}_{\mathrm{E}}$ shifts upward to $S_{\mathrm{Et}}$ and the effect of this change would be to lower enrollment to $\mathrm{Q}_{\mathrm{E} 2}$, raise tuition to $\mathrm{P}_{\mathrm{E} 2}$, and raise the subsidy to $\mathrm{S}_{2}$. The actual effect of increased urbanization on subsidies to higher education


Figure 6: The Effects of an Increase in Urbanization on Public Higher Education Equilibrium
is the combination of the above two and can be expressed mathematically. Let the degree of urbanization be $X_{5}$. Then, replacing $X_{i}$ with $X_{5}$ in equation (7.1) and substituting in the appropriate partials for the intercept terms with respect to urbanization gives

$$
\begin{equation*}
\frac{\partial S^{*}}{\partial X_{5}}=\frac{b+c}{a+b+c} \alpha_{5}+\frac{a}{a+b+c} \theta_{5}>0 \tag{22}
\end{equation*}
$$

A change in the degree of urbanization of a state on subsidy level is positive since $\alpha_{5}$ and $\theta_{5}$ are positive.

The effect of a change in the degree of urbanization on tuition can be seen to be ambiguous using equation (9.1).

$$
\begin{equation*}
\frac{\partial P_{E}^{*}}{\partial X_{5}}=-\frac{b}{a+b+c} \alpha_{5}+\frac{b}{a+b+c} \theta_{5} \geq 0 \tag{23}
\end{equation*}
$$

Enrollment is ambiguously affected by the degree of urbanization in a state as demonstrated in equation (24).

$$
\begin{equation*}
\frac{\partial Q_{\varepsilon}^{\dot{~}}}{\partial X_{5}}=\frac{\alpha_{5}-\theta_{5}}{a+b+c} \geq 0 \tag{24}
\end{equation*}
$$

## Population Density

Population density may influence the private demand curve for public higher education. As population density rises, the private demand curve for public higher education would rise since transportation costs to students would fall. If the change in population density only affected the private demand curve for public higher education, a new equilibrium (previously illustrated in Figure 5) would occur at $E_{1}$ where the new total demand curve $D_{\text {S1 }}$ intersects the supply curve for public higher education. The new equilibrium would have higher enrollment and tuition $Q_{E 1}$ and $P_{E 1}$ and a lower subsidy level $\mathrm{S}_{1}$.

Population density may also affect PVEB, the present value of the external benefits curve. As a state's population becomes increasingly dense, the benefits of higher education become more highly valued by the states' citizens. This implies that the individual valuation of the external benefits of higher education would rise, causing an upward shift in the IVEB curve and therefore a corresponding increase in the PVEB curve to PVEB $_{1}$. If this were the only change, the upward shift in the PVEB curve would result in an increase in the total demand curve for higher education from $D_{s}$ to $D_{S 1}$ and a new equilibrium point at $E_{1}$. The new equilibrium, previously illustrated in Figure 5, would have a higher enrollment of $Q_{E_{1}}$, a lower tuition of $P_{E_{2}}$, and a higher subsidy level of $S_{2}$.

The combined effects of the changes in PVEB and $D_{\mathrm{E}}$ from a change in population density are ambiguous with respect to subsidy and tuition and positive with regard to enrollment.

Let population density be denoted as $X_{6}$. Then, replacing $X_{i}$ with $X_{6}$ in equation (7.1) and substituting in the appropriate partials for the intercept terms with respect to population density gives

$$
\begin{equation*}
\frac{\partial S^{*}}{\partial X_{6}}=\frac{b+c}{a+b+c} \alpha_{6}-\frac{a}{a+b+c} \gamma_{6} \frac{\geq}{<} 0 . \tag{25}
\end{equation*}
$$

Because all of the parameters are positive, a change in the density of a state's population on the optimal subsidy level is ambiguous.

The effect of a change in the density of a state's population on tuition levels can be seen to be ambiguous with equation (26).

$$
\begin{equation*}
\frac{\partial P_{E}^{*}}{\partial X_{6}}=-\frac{b}{a+b+c} \alpha_{6}+\frac{a+c}{a+b+c} \gamma_{6} \geq 0 \tag{26}
\end{equation*}
$$

Population density has a positive effect on enrollment levels as seen in equation (27).

$$
\begin{equation*}
\frac{\partial Q_{E}^{-}}{\partial X_{6}}=\frac{\alpha_{6}+\gamma_{6}}{a+b+c}>0 \tag{27}
\end{equation*}
$$

## Increased Future Earnings

Numerous studies have found that expected increases in after-tax future earnings is a significant reason for individuals to enter college. Mattila found in his study of the determinants of male enrollments that the supply of males to higher education had an elasticity response to salaries that ranged from 86 to 1.39 (Mattila, 1982). Other authors have also found positive elasticities of varying magnitudes (Freeman, 1986).

The theory of human capital, developed by Becker, asserts that individuals invest in education which raises their productivity and leads to greater earnings. Individuals view education as an
investment and choose the level of education that yields the highest expected rate of return. The expected rate of return to education is "that percentage rate of return that discounts the stream of earnings expected by the student over his or her life cycle back to its present value and equates them to the total educational costs compounded forward to the date of graduation" (McMahon, 1987). As expected earnings rise, students choose to invest more in human capital than they might have due to the corresponding increase in the expected rate of return to college education. The appropriate measure for individuals is the post-tax earnings differential (Verry, 1987; Galper and Dunn, 1969). An expected increased earnings differential would lead to an upward shift in $\mathbf{D}_{\mathrm{E}}$, and as previously depicted in Figure 5, would result in an increase in enrollment, a decrease in subsidy and an increase in tuition.

At the same time, PVEB would also shift rightward given the marginal tax rate. States expect to realize higher tax collections at current rates with an expected increase in future incomes. In this case, the appropriate measure is the pre-tax earnings differential. Windham (1976) questions the legitimacy of this argument for supporting subsidization for higher education, however, conventional wisdom accepts this as a reasonable claim. As shown in Figure 5, this shift would increase enrollment, decrease tuition and increase subsidy levels.

The overall effect of an increase in expected after-tax future earnings is equivalent to an increase in average educational attainment which is to increase enrollment and indeterminate with respect to tuition and subsidy. This can be verified by equations (28) through (30). Recall that all of the parameters are greater than zero.

$$
\begin{gather*}
\frac{\partial S^{*}}{\partial X_{7}}=\frac{b+c}{a+b+c} \alpha_{7}-\frac{a}{a+b+c} \gamma_{7} \frac{\geq}{<} 0  \tag{28}\\
\frac{\partial P_{E}^{*}}{\partial X_{7}}=-\frac{b}{a+b+c} \alpha_{7}+\frac{a+c}{a+b+c} \gamma_{7} \geq 0  \tag{29}\\
\frac{\partial Q_{E}^{*}}{\partial X_{7}}=\frac{\alpha_{7}+\gamma_{7}}{a+b+c}>0 \tag{30}
\end{gather*}
$$

## Foregone Earnings

The decision to attend college is influenced heavily by the opportunities that a student faces upon completion of high school. Foregone earnings is one way to measure the opportunity cost of investing in a higher degree and is an important component of the expected rate of return calculation for higher education. Foregone earnings affect both the $\mathrm{D}_{\mathrm{B}}$ and PVEB curves but in the opposite direction as future earnings. As foregone earnings rise, the rate of return to a college education falls, resulting in some students being drawn into the labor market after high school. Manski and Wise found that local wage rates, a proxy for foregone earnings, negatively influenced the decision to apply for college admission (Manski and Wise, 1983). This could be shown by a leftward or downward shift in $\mathrm{D}_{\mathrm{E}}$ as shown in Figure 7. Figure 7 illustrates that as private demard for higher education falls, the total demand curve for higher education, $\mathrm{D}_{\mathbf{s}}$, also shifts left. The effect on the higher education equilibrium is falling enrollment and tuition to $\mathbf{Q}_{\mathrm{E} 1}$ and $\mathbf{P}_{\mathrm{E} 1}$ respectively and rising subsidy levels to $S_{1}$.


Figure 7: The Effects of an Increase in Foregone Earnings on Public Higher Education Equilibrium

Given current tax rates, an increase in foregone earnings will lead to less additional tax collections resulting from the education decision (Verry, 1987; Cohn, 1979). The loss in tax revenue would cause the PVEB curve to shift leftward and would lower enrollment to $\mathrm{Q}_{\mathrm{E} 1}$, raise tuition to $\mathbf{P}_{\mathrm{E} 2}$, and lower the subsidy to $\mathrm{S}_{\mathbf{2}}$ as demonstrated in Figure 7.

The combined effect of an increase in foregone earnings would be uncertain with respect to subsidy and tuition levels and would lead to lower enrollments. Let foregone earnings be $\mathbf{X}_{8}$. Then, substituting the partials of the intercept terms with respect to $X_{8}$ into equation (7.1) gives the effect of a change in foregone earnings on subsidy levels. Note that the intercepts are negatively influenced by an increase in foregone earnings; therefore these partials are negative.

$$
\begin{equation*}
\frac{\partial S^{*}}{\partial X_{8}}=\frac{b+c}{a+b+c} \alpha_{\mathrm{g}}-\frac{a}{a+b+c} \gamma_{8} \frac{\geq}{<} 0 \tag{31}
\end{equation*}
$$

An increase in foregone earnings has an indeterminate effect on subsidy levels.
Equation (32), derived from equation (9.1) demonstrates the ambiguity of an increase in foregone earnings on tuition levels.

$$
\begin{equation*}
\frac{\partial P_{E}^{*}}{\partial X_{8}}=-\frac{b}{a+b+c} \alpha_{8}+\frac{a+c}{a+b+c} \gamma_{8} \geq 0 \tag{32}
\end{equation*}
$$

The decrease in enrollment is readily apparent from Figure 7 and can be proven by applying equation (11.1). Replacing the partials of the intercept terms with respect to $X_{i}$ with the partials of the intercepts with respect to $\mathrm{X}_{8}$ gives

$$
\begin{equation*}
\frac{\partial Q_{E}^{*}}{\partial X_{\mathrm{B}}}=\frac{\alpha_{\mathrm{g}}+\gamma_{\mathrm{B}}}{a+b+c}<0 . \tag{33}
\end{equation*}
$$

The use of foregone earnings as a measure of the opportunity cost of investing in higher education has come under fire in recent years. Many students maintain part-time jobs during their college career and for the most part, these are equivalent to the type of jobs held by high school graduates (Cohn, 1979; Campbell and Seigel, 1967). Parsons found that foregone earnings as a measure of opportunity costs seriously underestimates the true student cost of education. His study showed that students reduce leisure time as well as work hours for school, therefore, foregone earnings are only a part of the opportunity cost of education. This is significant at the higher education level but is much more influential at the secondary level (Parsons, 1974).

## Marginal Tax Rate

Consider the case of an increase in the marginal tax rate. When the marginal tax rate rises, the private demand curve for higher education would shift downward since the present value of the after-tax component of the expected increase in future earnings will decline. The decrease in aftertax expected earnings would lower the expected private rate of return to a college education. The downward shift in $D_{E}$ seen in Figure 8 would lower $D_{s}$ to $D_{S 1}$ and would lead to a decrease in


Figure 8: The Effects of an Increase in the Marginal Tax Rate on Public Higher Education Equilibrium
enrollment, a decrease in tuition levels and an increase in the amount of per-student subsidy, $\mathbf{Q}_{\mathrm{E}}$, $P_{\text {E1 }}$, and $S_{1}$, respectively.

At the same time, the PVEB curve would shift up to PVEB $_{1}$ due to the increase in expected future tax collections and therefore, $D_{s}$ would shift up to $D_{s 2}$. This shift would, ceteris paribus, lead to an increase in enrollment, $\mathrm{Q}_{\mathrm{E} 2}$, a decrease in tuition, $\mathrm{P}_{\mathrm{E} 2}$, and an increase in the per-student subsidy, $\mathrm{S}_{2}$. Since the gain to tax revenue is equal to the loss to the taxpayer, the rise in PVEB and the fall in $D_{B}$ should be equal and the net effect on $D_{s}$ should be nil. Thus, enrollments should be unchanged, but subsidies should rise while tuition falls. Let the marginal tax rate be denoted by $\mathrm{X}_{\mathbf{g}}$. Then, using (7.1),

$$
\begin{equation*}
\frac{\partial S^{*}}{\partial X_{9}}=\frac{b+c}{a+b+c} \alpha_{9}-\frac{a}{a+b+c} \gamma_{9}>0 . \tag{34}
\end{equation*}
$$

Since $\alpha_{9}>0$ and $\gamma_{9}<0$, this partial derivative has a positive sign. Therefore, an increase in the marginal tax rate would lead to an increase in the optimal per-student subsidy.

Equation (35) confirms that a decline in tuition would occur from an increase in the marginal tax rate.

$$
\begin{equation*}
\frac{\partial P_{E}^{*}}{\partial X_{9}}=-\frac{b}{a+b+c} \alpha_{9}+\frac{a+c}{a+b+c} \gamma_{9}<0 \tag{35}
\end{equation*}
$$

The effect on enrollment that would occur from a change in the marginal tax rate can be seen in equation (36). If $\alpha_{9}$ and $\gamma_{9}$ are of equal values but with opposite signs, $\alpha_{9}+\gamma_{9}=0$ and the partial derivative of enrollment with regard to the marginal tax rate would be zero.

$$
\begin{equation*}
\frac{\partial Q_{E}^{*}}{\partial X_{9}}=\frac{\alpha_{9}+\gamma_{9}}{a+b+c} \frac{\geq}{<} 0 \tag{36}
\end{equation*}
$$

## Migration

Migration of college educated people is a concern for many states, particularly in the Midwest, as graduates relocate to other regions or states. The loss of educated labor means that some states are unable to recoup their investment in higher education while others experience an unpaid-for gain. States that have a high percentage of their college graduates who remain in the state would have increased incentive to fund higher education while states that have a high out migration rate are net losers and would have a lower incentive to fund higher education due to the loss of external benefits (McMahon, 1987).

An increase in the percentage of college graduates who remain in the state would cause the PVEB curve to shift right to PVEB $_{1}$ as seen in Figure 9 and would result in an increase in public enrollment to $Q_{E 1}$, a decrease in tuition to $P_{E 1}$, and an increase in optimal subsidy to $S_{1}$ due to the resulting rightward shift in $D_{s}$ to $D_{s 1}$. Let the percentage of college graduates who remain in the


Figure 9: The Effects of Lower Out-Migration on Public Higher Education Equilibrium
state be denoted as $\mathbf{X}_{10}$. Then the effects on tuition, subsidy, and enrollment can be confirmed mathematically with equations (37), (38), and (39).

An increase in the percentage of college graduates who remain in a state would lead to a higher optimal per-student subsidy as seen in equation (37). Recall that $\alpha_{10}$ is positive.

$$
\begin{equation*}
\frac{\partial S^{*}}{\partial X_{10}}=\frac{b+c}{a+b+c} \alpha_{10}>0 \tag{37}
\end{equation*}
$$

Equation (38) shows that tuition would fall when there is an increase in the percentage of college graduates who remain in a state.

$$
\begin{equation*}
\frac{\partial P_{E}^{E}}{\partial X_{10}}=-\frac{b}{a+b+c} \alpha_{10}<0 \tag{38}
\end{equation*}
$$

The increase in enrollment that would occur from an increase in the percentage of college graduates who remain in a state is seen in equation (39).

$$
\begin{equation*}
\frac{\partial Q_{E}^{\cdot}}{\partial X_{10}}=\frac{\alpha_{10}}{a+b+c}>0 \tag{39}
\end{equation*}
$$

## Outside Funding

Grants, contributions, endowment income, and federal aid may affect the location of the supply curve of higher education. Theoretically, it seems reasonable to expect that increases in outside sources of income would lower the supply curve of higher education as the cost-per-student is partially offset. Another reason why the supply curve could decline is that grant money covers a fraction of faculty salaries, therefore, the money that is no longer needed for faculty compensation could be used to hire graduate assistants for teaching which would lower instructional costs. Outside sources of income are predicted to lower the supply curve for higher education to $\mathrm{S}_{\mathrm{EI}}$ as presented in


Figure 10: The Effects of an Increase in Outside Funding on Public Higher Education Equilibrium

Figure 10. The rightward shift of $S_{E}$ would result in a new equilibrium point $E_{1}$ with a lower subsidy level of $\mathrm{S}_{\mathbf{1}}$, lower tuition level of $\mathrm{P}_{\mathrm{EL}}$, and higher enrollment level of $\mathrm{Q}_{\mathrm{El}}$. Let outside money be denoted as $\mathrm{X}_{11}$. The predicted effects of outside money on the subsidy, tuition, and enrollment levels can be shown mathematically. Note that $\theta_{11}$ is negative.

$$
\begin{equation*}
\frac{\partial S^{*}}{\partial X_{11}}=\frac{a}{a+b+c} \theta_{11}<0 \tag{40}
\end{equation*}
$$

As expected, the optimal subsidy level would fall with outside funding.
(41)

$$
\frac{\partial P_{E}^{*}}{\partial \mathrm{X}_{11}}=\frac{b}{a+b+c} e_{11}<0
$$

Equation (41) demonstrates that tuition levels would fall with outside money.

$$
\begin{equation*}
\frac{\partial Q_{E}^{*}}{\partial X_{11}}=-\frac{\theta_{11}}{a+b+c}>0 \tag{42}
\end{equation*}
$$

An increase in outside money would result in an increase in enrollments as shown by equation (42).

An alternative view regarding outside money is posited by Bowen. Bowen asserts in his "revenue theory of costs" that "an institution raises all it can and spends all it raises" (Bowen, 1987). The consequences of his theory is that the cost-per-student could actually rise as institutions find new projects on which to spend the accumulated funds. This behavior implies that the supply curve for higher education would rise with outside monies resulting in lower enrollments, higher tuition and higher subsidy levels. The empirical results may give some insight about the effects of outside funding sources on the cost curve of higher education.

## Junior College Enrollment

The existence and increasing importance of 2 -year public institutions has definite consequences for the higher education market. The 1991-92 fiscal year has seen many states decreasing or only marginally increasing total appropriations for higher education with an increasing priority for community colleges. Thirty-four states had an average two-year percentage gain of 3 percent for total state support of higher education with community colleges averaging a 13 percent two-year percentage gain (Jaschik, 1991).

Community colleges are included as part of the public higher education market and are therefore included in the subsidy, tuition and enrollment variables for the public sector. The percentage of public students that attend a junior college can be included in the analysis to give an indication as to the state governing board's philosophy towards the junior college option to meet enrollment needs. The existence of two-year public schools and enrollments would result in a cost savings to the state since it is less expensive to educate students in that manner. This would imply a decline or rightward shift of the supply curve for public higher education because the average cost-per-student falls for the state. The lowering of the supply curve causes the optimal subsidy and tuition to fall and overall enrollment to rise as seen in the preceding graph, Figure 10.

Let the percentage of students enrolled in public junior colleges be represented by $\mathrm{X}_{12}$. Then the effect of junior college enrollments on the optimal subsidy, tuition, and enrollment levels can be
seen with equations (43), (44), and (45).

$$
\begin{equation*}
\frac{\partial S^{*}}{\partial X_{12}}=\frac{a}{a+b+c} \theta_{12}<0 \tag{43}
\end{equation*}
$$

Since $\theta_{12}<0$, it is clear that the optimal subsidy per-student falls with an increase in junior college enrollment as a percentage of total public enrollment.

$$
\begin{equation*}
\frac{\partial P_{E}^{*}}{\partial X_{12}}=\frac{b}{a+b+c} \theta_{12}<0 \tag{44}
\end{equation*}
$$

Tuition levels fall with an increase in the percentage of junior college enrollment.

$$
\begin{equation*}
\frac{\partial Q_{E}^{*}}{\partial X_{12}}=-\frac{\theta_{12}}{a+b+c}>0 \tag{45}
\end{equation*}
$$

Total public enrollments rise with an increase in the percentage of junior college enrollments in a state.

## Private School Orientation

The effects of the demand for private schooling on public higher education warrant further examination. While the vast majority of states have private education opportunities, there is a wide disparity in the scope of private schools. Historically, Massachusetts has had a long tradition of being predominantly private while Wyoming is characterized by 100 percent public schools. The private and public markets for higher education and the external benefits curve can be graphically represented as follows.

Figure 11 illustrates that the external benefits curve, PVEB, is dependent upon total higher education enrollment in a state. As total enrollment rises there is a lower valuation for each additional student. Thus, the optimal per-student subsidy for public students is directly affected by


Total Enrollment



Figure 11: The Higher Education Markets with the External Benefits Curve
the level of private enrollment in a state since it is likely that there is a similar valuation of the external benefits of private and public higher education. These three graphs can be combined into one graph as in Figure 12 to demonstrate the link between the private and public higher education markets due to the external benefits curve.

The supply and demand curves for private higher education are denoted as $S_{v}$ and $D_{v}$ with equilibrium enrollment and tuition of $\mathrm{Q}_{\mathrm{v}}{ }^{\prime}$ and $\mathrm{P}_{\mathrm{v}}{ }^{\circ}$. The public higher education market can be incorporated into this graph by using $\mathrm{Q}_{\mathrm{v}}{ }^{\text {as }}$ its origin point. The vertical line beginning at $\mathrm{Q}_{\mathrm{v}}{ }^{\circ}$ provides the vertical axis for the public higher education market. Therefore, the demand and supply curves for the public higher education markets, $D_{E}$ and $S_{E}$, have intercept terms, (B) and (C), at this point.

The external benefits curve, PVEB, has an intercept point of (A) in the private market and an intercept point of $\left(A-\mathrm{aQ}_{\mathrm{v}}{ }^{\circ}\right)$ in the public market. The two intercept terms reflect the external


Figure 12: Equilibrium in the Two-Sector Higher Education Market
benefits curve's dependence on total enrollment. The intercept in the public market, $\left(A-a Q_{V}{ }^{\circ}\right)$, is higher when there is low private enrollment and lower when the converse is true. This relationship can be expressed as a partial derivative, $\partial\left(A-a Q_{v}\right) / \partial Q v=-a$.

As previously established, equilibrium in the public higher education market is determined by the intersection of the public education supply curve, $S_{E}$, and the total demand curve for public higher education, $D_{s}$. The total demand curve for public higher education is the vertical summation of the private demand curve for public higher education and the external benefits curve, PVEB. Therefore, $D_{s}$ has an intercept of $\left[\left(A-a Q_{v}{ }^{\circ}\right)+B\right]$. The public higher education market equilibrium is one with enrollment $Q_{E}{ }^{*}$, tuition $P_{E}{ }^{\circ}$, and per-student subsidy $S^{*}$.

Figure 13 separates the two higher education markets and incorporates the PVEB curve into the public higher education market graph. The PVEB intercept in the public market is again (A$a Q_{v}{ }^{*}$. This graphical representation facilitates the discussion of the effects of an increase in private enrollments on the public higher education equilibrium.


Figure 13: The Private and Public Markets for Higher Education

States that are more privately oriented will have a higher private demand curve for private education. The higher private enrollments affect the public education market in several ways.

First, the external benefits curve, PVEB, would shift left in the public market reflecting the lower external valuation of each additional student due to increased private enrollment. The effect of the lower PVEB curve in the public market would be to decrease public enrollment and subsidy and to increase public tuition.

The second effect of increased private enrollments would be to shift the demand for public education leftward. Some of the students who would have been publicly educated are drawn away from the public education market toward private education. This implies that (B), the intercept of the private demand curve for public education, is negatively influenced by the quantity of private education, $\mathrm{Q}_{\mathbf{V}}$. If the quantity of private education is variable $\mathrm{X}_{13}$ in equation (2), then this relationship is represented by the partial of $B$ with respect to $Q_{V}$, which is $\gamma_{13}$. The leftward shift in
$D_{B}$ would result in a corresponding leftward shift in $D_{S}$ and would lead to lower public enrollment, higher subsidy and lower tuition.

The private and public education markets can be represented mathematically.

Public education
$D_{E}: P_{E}{ }^{D}=B \cdot b Q_{E}$
$S_{B}: P_{B}{ }^{s}=C+Q_{B}$
PVEB: $S=A-a Q$
Equilibrium: $P_{E}^{D}+S=P_{E}{ }^{\mathbf{S}}$

Private education
$D_{\mathbf{v}}: \mathbf{P}_{\mathbf{v}}{ }^{\mathrm{D}}=\mathbf{W}-w \mathbf{Q}_{\mathbf{v}}$
$S_{v}: P_{v}^{s}=\mathbf{U}+u Q_{v}$

Equilibrium: $\mathbf{P}_{\mathbf{v}}{ }^{\mathbf{S}}=\mathbf{P}_{\mathbf{v}}{ }^{\mathbf{D}}$
$Q_{X}$ and $Q_{V}$ are public and private enrollment respectively, $Q$ is the sum of the two enrollment levels and $P_{v}$ represents private tuition. $W, U, w$, and $u$ are positive parameters. Again, $A, B$, and $C$ are intercepts which are linearly related to the explanatory variables. As with public education, the intercept terms for the private education supply and demand curves, $W$ and $U$, are functions of some exogenous variables.

The optimal solutions are:

$$
\begin{equation*}
Q_{v}^{*}=\frac{W-U}{w+u} \tag{46}
\end{equation*}
$$

$$
\begin{equation*}
Q_{E}^{*}=\frac{A+B-C-a\left(\frac{W-U}{w+u}\right)}{a+b+c} \tag{48}
\end{equation*}
$$

$$
\begin{equation*}
P_{E}^{*}=B-b\left[\frac{A+B-C-a\left(\frac{W-U}{w+u}\right)}{a+b+c}\right] \tag{49}
\end{equation*}
$$

$$
\begin{equation*}
S^{*}=A-a\left(\frac{W-U}{W+u}\right)-a\left[\frac{A+B-C-a\left(\frac{W-U}{W+u}\right)}{a+b+c}\right] \tag{50}
\end{equation*}
$$

The effects of private enrollment on public enrollment, tuition and subsidy can be derived using equations (46) through (50). Recall that $B=\sum_{i=1}^{n} \gamma_{i} X_{i}$. Let $Q_{v}{ }^{\circ}$ be denoted as $X_{13}$. Then, replacing B in equations (48), (49), and (50), the effect of private enrollments on the optimal subsidy, public tuition and public enrollments can be easily derived. Note that

$$
\frac{\partial B}{\partial X_{13}}=\gamma_{13}<0 \text { and that } Q_{V}^{*}=\frac{W-U}{w+u}
$$

Regarding the optimal subsidy,

$$
\begin{gather*}
\frac{\partial S^{*}}{\partial X_{13}}=\frac{\partial}{\partial X_{13}}\left[A-a\left(\frac{W-U}{W+u}\right)-\frac{a}{a+b+c}(A-C)-\frac{a}{a+b+c}\left(\sum_{1}^{12} \gamma_{1} X_{i}+\gamma_{13} X_{13}+\sum_{14}^{n} \gamma_{t} X_{i}\right)+\frac{a^{2}}{a+b+c}\left(\frac{W-U}{w+u}\right)\right] \\
\frac{\partial S^{*}}{\partial X_{13}}=\frac{\partial}{\partial X_{13}}\left[A-a Q_{V}^{*}-\frac{a}{a+b+c}(A-C)-\frac{a}{a+b+c}\left\{\sum_{1}^{12} \gamma_{1} X_{i}+\gamma_{13} Q_{V^{+}}^{*} \sum_{14}^{n} \gamma_{r} X_{i}\right\}+\frac{a^{2}}{a+b+c} Q_{V}^{*}\right] \\
51)  \tag{51}\\
\frac{\partial S^{*}}{\partial X_{13}}=-a-\frac{a \gamma_{13}}{a+b+c}+\frac{a^{2}}{a+b+c} \geq 0 .
\end{gather*}
$$

Therefore the effect of private enrollments on the optimal subsidy to public education is uncertain.
The effect of private enrollments on public tuition can be derived using equation (49) and substituting $\sum_{i=1}^{n} \gamma_{i} X_{i}$ for $B$.

$$
\begin{gathered}
\frac{\partial P_{E}^{*}}{\partial X_{13}}=\frac{\partial}{\partial X_{13}}\left[\frac{a+c}{a+b+c}\left(\sum_{1}^{12} \gamma_{i} X_{i}+\gamma_{13} X_{13}+\sum_{14}^{n} \gamma_{i} X_{i}\right)-\frac{b}{a+b+c}(A-C)+\frac{a b}{a+b+c}\left(\frac{W-U}{w+u}\right)\right] \\
\frac{\partial P_{E}^{*}}{\partial X_{13}}-\frac{\partial}{\partial X_{13}}\left[\frac{a+c}{a+b+c}\left(\sum^{12} \gamma_{i} X_{i}+\gamma_{13} Q_{V}^{*}+\sum^{n} \gamma_{i} X_{t}\right)-\frac{b}{a+b+c}(A-C)+\frac{a b}{a+b+c} Q_{V}^{*}\right]
\end{gathered}
$$

$$
\begin{equation*}
\frac{\partial P_{E}^{*}}{\partial X_{13}}=\frac{a+c}{a+b+c} \gamma_{13}+\frac{a b}{a+b+c} \geq 0 \tag{52}
\end{equation*}
$$

The effect of private enrollments on public tuition is also indeterminate.
Regarding public enrollment,

$$
\begin{aligned}
& \frac{\partial Q_{E}^{*}}{\partial X_{13}}=\frac{\partial}{\partial X_{13}}\left[\frac{A-C}{a+b+c}+\frac{1}{a+b+c}\left(\sum_{1}^{12} \gamma_{i} X_{i}+\gamma_{13} X_{13}+\sum_{14}^{n} \gamma_{i} X_{i}\right)-\frac{a}{a+b+c}\left(\frac{W-U}{w+u}\right)\right] \\
& \frac{\partial Q_{E}^{*}}{\partial X_{13}}=\frac{\partial}{\partial X_{13}}\left[\frac{A-C}{a+b+c}+\frac{1}{a+b+c}\left(\sum^{12} \gamma_{1} X_{i}+\gamma_{13} Q_{V}^{*}+\sum^{n} \gamma_{r} X_{i}\right)-\frac{a}{a+b+c} Q_{V}^{*}\right]
\end{aligned}
$$

$$
\begin{equation*}
\frac{\partial Q_{E}^{*}}{\partial X_{13}}=\frac{\gamma_{13}}{a+b+c}-\frac{a}{a+b+c}<0 . \tag{53}
\end{equation*}
$$

As expected, the effect of private enrollment on public enrollment is negative.

## State Appropriations to Private Institutions

Consider now the case where private schools are subsidized by the state. Let the intercept for the supply curve of private higher education, $U$, be equal to $T-G$ where $G$ is state appropriations and tuition grants to private schools and is greater than or equal to zero. There are many states that provide some type of financial assistance to private education.

Now suppose that the grant or endowment is positive so that costs for students decline in the private sector. This could be shown by a rightward shift, or drop, of the private supply curve. Then enrollment will rise in the private sector and again, assuming that the elasticity of substitution
between the two markets is not zero, the private demand curve for public education would shift left as some students are drawn away from the public education market. In this case it is convenient to consider the relationship between private price and the private demand for public education rather than the private enrollment since the grant directly lowers the private tuition. Therefore, the intercept, B, of the private demand curve for public education is positively influenced by the price of private education, $\mathbf{P}_{\mathbf{v}}$. If the price of private education is variable $\mathbf{X}_{16}$, then this relationship is represented by the partial derivative of $B$ with respect to $\mathbf{P}_{\mathbf{v}}$, which is $\boldsymbol{\gamma}_{14}$.

The increase in private enrollments from the decrease in the cost has the effect of lowering the PVEB curve in the public education market since, as discussed previously, the height of the PVEB curve in the public education market is dependent upon the level of private enrollment.

The price of private education declines and private enrollment rises with a grant to private schools. It is also known that if the private demand curve for public education shifts back and to the left, and the PVEB curve shifts back and to the left, the total demand curve, $D_{s}$, also shifts left, resulting in a decline in enrollment in public institutions. However, it is not readily apparent whether the optimal per-student subsidy rises and whether public tuition rises when this leftward shift in $\mathbf{D}_{\mathbf{s}}$ occurs. These questions can be addressed with the mathematical comparative static techniques used previously.

Let the private school supply curve intercept, $U$, be equal to $T-G$, where $G$ is the state appropriations to private institutions and is greater than or equal to zero. Substituting T-G for $\mathbf{U}$, the solutions can be rewritten in the following manner:

$$
\begin{gather*}
Q_{V}^{*}=\frac{W-T+G}{w+u}  \tag{54}\\
P_{V}^{*}=T-G+\frac{u}{w+u}(W-T+G) \tag{55}
\end{gather*}
$$

(56)

$$
Q_{E}^{*}=\frac{A+B-C-a\left(\frac{W-T+G}{W+\mu}\right)}{a+b+c}
$$

$$
\begin{equation*}
P_{E}^{*}=B-b\left[\frac{A+B-C-a\left(\frac{W-T+G}{W+u}\right)}{a+b+c}\right] \tag{57}
\end{equation*}
$$

$$
\begin{equation*}
S^{*}=A-a\left(\frac{W-T+G}{W+u}\right)-\frac{a}{a+b+c}\left[A+B-C-a\left(\frac{W-T+G}{w+u}\right)\right] . \tag{58}
\end{equation*}
$$

The effect of an increase in $\mathbf{G}$ on the public education variables and the optimal subsidy can be derived in the same manner as in the preceding analysis. Let $\mathrm{P}_{\mathrm{v}}{ }^{\circ}$ be $\mathrm{X}_{14}$ so that $\partial(\mathrm{B}) / \partial\left(\mathrm{P}_{\mathrm{v}}{ }^{*}\right)=$ $\boldsymbol{\gamma}_{14}$. Substituting in the solution for $\mathrm{P}_{\mathbf{v}}{ }^{\cdot}$ from equation (55) for $\mathrm{X}_{14}$ in the expression for B gives
(59)

$$
B=\left[\begin{array}{lc}
13 & n \\
\sum_{i=1} \gamma_{i} x_{i}+\gamma_{14} P_{V}^{*}+\sum_{i=15} \gamma_{i} x_{i}
\end{array}\right]
$$

$$
B=\left[\begin{array}{lc}
13 & n \\
\sum_{i=1} \gamma_{i} X_{i}+\gamma_{14}\left\{T-G+\frac{u}{w+u}(W-T+G)\right\}+\sum_{i=15} \gamma_{i} X_{i}
\end{array}\right]
$$

Then, replacing B with (59) in equations (56), (57), and (58), the effect of a grant to private education on the public education variables and the optimal subsidy is easily derived.

The influence of state appropriations and assistance to private institutions has a negative effect on public enrollments.

$$
\begin{aligned}
& Q_{E}^{*}=\left[\begin{array}{c}
13 \\
A-C+\sum_{i-1} \gamma_{1} X_{i}+\gamma_{14}\left(T-G+\frac{u}{w+u}(W-T+G\}\right)+\sum_{i-15}^{n} \gamma_{1} X_{i} \\
\frac{\partial Q_{E}^{*}}{\partial G}=\left[-\frac{\gamma_{14}}{a+b+c}+\frac{\gamma_{14}\left(\frac{u}{w+u}\right)}{a+b+c}-\frac{a}{(a+b+c)(w+u)}\left(\frac{W-T+G}{w+u}\right)\right] \\
(60) \quad \\
\frac{\partial Q_{E}^{*}}{\partial G}=-\frac{w \gamma_{14}-a}{(a+b+c)(w+u)}<0
\end{array}\right.
\end{aligned}
$$

Regarding public tuition, the effect of private institution subsidization is ambiguous.

$$
P_{E}^{\dot{e}}=\frac{a+c}{a+b+c}\left[\begin{array}{l}
13 \\
\left.\sum_{i-1} \gamma_{X} X_{i}+\gamma_{14}\left\{T-G+\frac{u}{w+u}(W-T+G)\right\}+\sum_{i=15}^{n} \gamma_{i} X_{i}\right]
\end{array}\right]
$$

(61)

$$
\begin{aligned}
& -\frac{b(A-C)}{a+b+c}+\frac{a b}{a+b+c}\left(\frac{W-T+G}{w+u}\right) \\
\frac{\partial P_{E}^{*}}{\partial G} & =\frac{a+c}{a+b+c}\left[\gamma_{14}\left(-1+\frac{u}{w+u}\right)\right]+\frac{a b}{a+b+c}\left(\frac{1}{w+u}\right) \\
\frac{\partial P_{E}^{*}}{\partial G} & =\frac{-(a+c) \gamma_{14} w+a b}{(a+b+c)(w+u)} \geq 0
\end{aligned}
$$

The effect of state funding for private institutions on the optimal subsidy for public students is ambiguous.

$$
\begin{align*}
S^{*} & =A-a\left(\frac{W-T+G}{w+u}\right)-\frac{a}{a+b+c}(A-C) \\
& -\frac{a}{a+b+c}\left[\sum_{i=1}^{13} \gamma_{i} X_{i}+\gamma_{14}\left(T-G+\frac{u}{w+u}(W-T+G)\right)+\sum_{i=15}^{n} \gamma_{i} X_{i}\right]+\frac{a^{2}}{a+b+c}\left(\frac{W-T+G}{w+u}\right)  \tag{62}\\
\frac{\partial S^{*}}{\partial G} & =-\frac{a}{w+u}-\frac{a}{a+b+c}\left[\gamma_{14}\left(\frac{-w}{w+u}\right)\right]+\frac{a^{2}}{(a+b+c)(w+u)} \\
\frac{\partial S^{*}}{\partial G} & =\frac{a \gamma_{14} w-a(b+c)}{(a+b+c)(w+u)} \frac{\geq}{<} 0
\end{align*}
$$

The effects of the grant on the price of public education and the per-student subsidy appear to be ambiguous and warrant further examination. If the demand for higher education is considered as a whole and not separated between private and public enrollment, then it follows from the Law of Demand that a decline in price results in an increase in quantity demanded. This fact implies that

$$
\left|\frac{\partial Q_{V}^{*}}{\partial G}\right|>\left|\frac{\partial Q_{E}^{*}}{\partial G}\right|
$$

because if these two were equal, a decline in price would result in the same quantity demanded, requiring a vertical aggregated demand curve. The restriction on $\partial Q_{E} / \partial G$ and $\partial Q_{V} / \partial G$ is equivalent to
(63)

$$
\begin{gathered}
\left|\frac{\partial Q_{V}^{*}}{\partial G}\right|>\left|\frac{\partial Q_{E}^{*}}{\partial G}\right|=\frac{\partial Q_{V}^{*}}{\partial G}>-\frac{\partial Q_{E}^{*}}{\partial G} \\
\frac{1}{w+u}>-\left(-\frac{w \gamma_{14}+a}{(w+u)(a+b+c)}\right) \\
\frac{1}{w+u}>\frac{w \gamma_{14}+a}{(w+u)(a+b+c)} \quad 1>\frac{w \gamma_{14}+a}{a+b+c} \\
a+b+c>w \gamma_{14}+a
\end{gathered} \quad \therefore \frac{b+c}{w}>\gamma_{14} .
$$

Applying this restriction to the partial of the subsidy with respect to the grant gives:

$$
\begin{align*}
\frac{\partial S^{*}}{\partial G} & =\frac{a \gamma_{14} w-a(b+c)}{(a+b+c)(w+u)} \geq 0 \text { substituting in } \gamma_{14}=\frac{b+c}{w} \\
& =\frac{a\left(\frac{b+c}{w}\right) w-a(b+c)}{(a+b+c)(w+u)}  \tag{64}\\
& =\frac{a(b+c)-a(b+c)}{(a+b+c)(w+u)}=0 \text { since } \gamma_{14}<\frac{b+c}{w} \quad \frac{\partial S^{*}}{\partial G}<0 .
\end{align*}
$$

Therefore, the effect of an increase in the level of a grant to private schools is a decrease in the optimal subsidy to public students.

Substituting the restriction into the partial derivative of public schools' price gives:

$$
\left.\begin{array}{l}
\begin{array}{rl}
\frac{\partial P_{E}^{*}}{\partial G} & =\frac{a b-\gamma_{14}(w)(a+c)}{(w+u)(a+b+c)} \geq 0 \text { substituting in } \gamma_{14}=\frac{b+c}{w} \\
& =\frac{a b-\frac{b+c}{w}(a+c) w}{(w+u)(a+b+c)}=\frac{a b-(b+c)(a+c)}{(w+u)(a+b+c)}=\frac{-a c-b c-c^{2}}{(w+u)(a+b+c)}-\frac{-c}{w+u}<0
\end{array} \\
\text { since } \gamma_{14}<\frac{b+c}{w} \tag{65}
\end{array}\right\}
$$

The effect of a grant to private schools on public schools' price to students is negative.
The preceding pages explored the effects of changes in those exogenous variables that are expected to vary across states; this was done from a one-sector perspective of the higher education market in a state. The development of the theory in this way allowed for some interesting predications about the influence of the explanatory variables on the optimal per-student subsidy, tuition, and enrollment in the public higher education market. The next chapter describes the data used to test the theoretical predictions. Chapter 5 describes the empirical model used to test the one-sector theory and presents the results.

## CHAPTER 4. THE DATA

## Description of the Data

The cross-sectional data used in this analysis stems primarily from three sources, the Integrated Postsecondary Education Data System (IPEDS), the 1990 Statistical Abstract of the United States, and the 1980 Census of the Population.

The IPEDS data is collected by the National Center for Education Statistics (NCES) by annual surveys to all recognized and accredited postsecondary institutions in the U.S. The IPEDS surveys replace the Higher Education General Information Surveys (HEGIS) and are for the 1988-1989 fiscal year. Two surveys were used for this analysis, the enrollment survey and the finance survey. The institutional data included were aggregated to the state level only if enrollment and financial data were complete or could be determined and if they were for corresponding institutions. The data set excludes all proprietary institutions and institutions in U.S. territories. All public institutions of higher education, both two-year and four-year and above, were included as well as all two-year and four-year and above private institutions.

The higher education variables from the enrollment survey include total full-time undergraduate and post-baccalaureate students, total part-time undergraduate and post-baccalaureate students for the four-year and above institutions and the total full- and part-time undergraduate students at twoyear institutions. The enrollment figures were converted to full-time equivalent (FTE) students according to the following standard formulation provided by the National Center for Higher Education Management Systems. The formula for full-time equivalency is: FTE $=$ full-time students +.33 (part-time students).

The data included in the finance survey is subject to a greater level of measurement error than is the enrollment survey data due to the vast differences among the higher education institutions and the differences in interpretation of the variables included in the survey. The financial data also include imputed data that may not accurately reflect the non-reporting institutions's financial
characteristics and may tend to underestimate the actual variation that exists in the data. There were many instances in the private sector where the state appropriations variable and other financial variables were missing in the data tape. This required making phone calls to the appropriate governing board. In all cases, the missing values were supposed to be zeros reflecting the fact that the state did not appropriate funds for operating expenses of the private institutions. The IPEDS survey data is not ideal; however, it is the most comprehensive for the nation as a whole and, therefore, is the best source available for this type of cross-sectional analysis. Special programs such as tuition grant programs for private institutions are included in the analysis with the use of data from the National Association of State Scholarship and Grant Programs.

The demographic variables for the states were collected from the 1990 U.S. Statistical Abstract and the 1980 Census of the Population when 1988 data were not available. Several measures were created from the Census and Statistical Abstract variables to fit the needs and purpose of this analysis. These included foregone earnings, the increase in expected future earnings, the marginal tax rate, and the net migration index.

The variable for foregone earnings of a college student is the weighted average of the log mean earnings of male and female high school graduates, ages 18-24, working 35 or more hours a week. This measure is a reasonable proxy for the loss in income from attending college directly after high school and was converted to logs to correspond to the expected increase in future earnings variable. The data for this measure was collected from the 1980 Census.

The increase in expected future earnings was calculated as the difference between the log mean earnings of college graduates and the log mean earnings of high school graduates at ages 25-34 years. This proxy was chosen based on Mincer's short-cut method which hypothesizes that relative earnings is useable as a measure of the marginal rate of return to college versus high school at about 10 years of work experience. Mincer's short-cut method relies on the "overtaking concept"
which is that at the overtaking level of experience, the present value of the lifetime earnings stream from investing in additional schooling can be approximated by current earnings.

Let average high school graduate earnings evolve according to the earnings function $\mathbf{Y}_{\boldsymbol{s}}=\phi$ $(s, x)$ where $s$ is the number of years of schooling and $x$ is the number of years of experience. Let $V_{\text {, }}$ be the present value of this earnings profile.

$$
V_{s}=\int_{0}^{n} \phi(s, x) e^{-n(a+x)} d x
$$

where $r$ is the discount rate.
Let $\overline{\mathrm{Y}}$, be a constant level of earnings which has the same present value as the earnings profile for high school graduates. Define the overtaking level of experience as $\mathbf{x}^{\dot{\prime}}(\mathrm{s})$ such that $\overline{\mathbf{Y}}_{\mathbf{1}}=\boldsymbol{\phi}$ $\left(s, x^{\prime}(s)\right)$. That is, $x^{\prime}(s)$ is the level of experience where the annuity value of the high school graduate's lifetime earnings stream, $\mathrm{V}_{\mathrm{b}}$, is equal to $\overline{\mathrm{Y}}_{\mathrm{g}}$, a constant level of earnings. Then $\mathrm{V}_{\mathbf{1}}$ can be written as

$$
V_{s}=\int_{0}^{n} \bar{y}_{s} e^{-r x d x} d x=\frac{\alpha \bar{y}_{s}}{r}=\frac{\alpha \phi\left(s, x^{x}(s)\right)}{r}
$$

where $\alpha=\left(1-e^{-m}\right)$. This takes advantage of the fact that at the overtaking point $x^{\prime}(\mathrm{s})$,

$$
\begin{aligned}
\bar{y}_{s} & =\phi\left(s, x^{*}(s)\right)=\int_{0}^{n} \bar{y}_{s} e^{-r x} d x=\bar{y}_{s}\left(-\frac{1}{r} e^{-m}\right)+C b_{b}^{n} \\
& =\bar{y}_{s}\left(-\frac{1}{r} e^{-m}+\frac{1}{r}\right)=\frac{\bar{y}_{s}}{r}\left(1-e^{-m}\right)
\end{aligned}
$$

In a similar manner the earnings function for college graduates can be defined.
Let $\mathbf{Y}_{\mathrm{s}+\mathrm{d}}=\phi(\mathrm{s}+\mathrm{d}, \mathrm{x})$ be the earnings function for college graduates with a present value of $\mathrm{V}_{\mathrm{s}+\mathrm{d}}$.

$$
V_{s+d}=\int_{0}^{n} \phi(s+d x) e^{-r(s+d x)} d x
$$

Let $\bar{Y}_{t+d}$ be a constant level of earnings with the same present value as $V_{t+d \cdot} \mathbf{x}^{0}(s+d)$ is the overtaking level of experience for the earnings profile of college graduates such that $\bar{y}_{s+d}-\phi(s+d x *(s+d))$. Then $V_{t+d}$ can be written in the following manner

$$
V_{s+d}=\int_{0}^{n} \bar{y}_{s+d} e^{-r x} d x=\frac{\alpha \bar{x}_{s+d}}{r}=\frac{\alpha \phi\left(s+d x^{*}(s+d)\right)}{r}
$$

where $\alpha=1-e^{-\pi}$.
Then, let $\hat{\rho}$ be an estimate of the internal rate of return to an individual with $s$ years of schooling who invests an additional d years. By definition, $\hat{\rho}$ is the rate of discount that solves $(1+\rho)^{d} V_{\mathbf{t}}=V_{s+d}$ or alternatively,

$$
\begin{aligned}
& (1+\rho)^{d}=\frac{V_{s+d}}{V_{s}}=\frac{\frac{\alpha\left\{\phi\left(s+d x^{*}(s+d)\right)\right\}}{r}}{\frac{\alpha\left\{\phi\left(s, x^{*}(s)\right)\right\}}{r}} \\
& (1+\rho)^{d}=\frac{\phi\left(s+d x^{*}(s+d)\right\}}{\phi\left\{s, x^{*}(s)\right\}}
\end{aligned}
$$

Taking logs,

$$
\begin{aligned}
d \ln (1+\rho) & =\ln \left\{\phi\left(s+d x^{*}(s+d)\right)\right\}-\ln \left\{\phi\left(s x^{*}(s)\right)\right\} \\
\rho & =\frac{1}{d}\left\{\ln V_{s+d}-\ln V_{s}\right\}=\frac{1}{d}\left\{\ln \bar{y}_{s+d}-\ln \bar{y}_{s}\right\} .
\end{aligned}
$$

This expression provides a shortcut method for estimating the internal rate of return if the two overtaking levels of experience are known. Solving for $\hat{\rho}$ can be achieved by inserting the average log earnings levels of college and high school graduates at their overtaking levels into the above equation. Mincer argues that this can be approximated at 10 years of experience and that the
overtaking level of experience will be less than or equal to the reciprocal of the internal rate of return. It is not necessary to divide by the increase in the number of years of school, in this case four, since dividing by a constant is a linear transformation of the variable. The following diagram illustrates the shortcut method of estimating the internal rate of return of investing in additional schooling.


Figure 14. Mincer's Shortcut Method for Estimating the Internal Rate of Return to Schooling

The marginal tax rate measure for each state is approximated by total own-source tax revenue for a state in 1988 divided by the 1988 Gross State Product.

The net migration index is a created measure that approximates the percentage of college graduates who remain in a state net of migration. The index attempts to isolate the effects of migration by comparing the percentage of people in a state who are college educated to the college participation rate in a state.

The net migration index equals the percent of college educated citizens in state j divided by the percent of college educated citizens in the U.S. in the numerator and the college participation rate in state j divided by the college participation rate in the U.S. in the denominator. The absence of a good measure of current migration necessitates the use of this measure which reflects past migration. However, this is not a serious drawback since the migration index is used as a measure of taxpayers' perceptions of migration.

The migration measure has a population weighted mean of 1.005 and ranges from .562 to 2.21 . The manner in which this index is calculated suggests that 1.00 is equivalent to zero net migration. Thus, a state that has a migration value near 1.00 is either retaining its college graduates or recouping those graduates that are lost due to out-migration.

Consider the migration values associated with Alabama and Alaska, .562 and 2.145 respectively. Alabama is experiencing net out-migration. The migration value of .562 implies that the state of Alabama loses nearly half of it's expected college graduates after migration has occurred. Alaska, with a 2.145 migration measure, benefits from a substantial net in-migration of college graduates from other states. Alaska, therefore, has double the expected number of college graduates relative to the college participation rate in the state.

## Definitions of the Variables

The educational variables used in the analysis are defined in the same manner as the IPEDS survey unless some aggregation was performed as in the case of state appropriations and outside monies to educational institutions.

State appropriations equals all amounts received by the higher education institutions through acts of a legislative body, except grants and contracts. These funds are for meeting current operating expenses, not for specific programs or projects. Also included are state need-based awards to undergraduates in public institutions. The dependent variable used in the regression
analysis of subsidies is calculated by dividing the sum of state appropriations and need-based aid by FTE enrollment in public institutions.

Tuition equals all tuition and fees, including student activity fees, assessed against students for education purposes. This figure includes tuition and fee remissions and exceptions and excludes room, board and other services rendered by auxiliary enterprises. The average-tuition-per-student variable in the regressions is tuition divided by FTE enrollment.

Enrollment is FTE enrollment figures for the various institutional types under consideration.
Private Orientation equals the FTE private enrollment in a state divided by the total FTE higher education enrollment in a state.

Private Appropriations equals all amounts received by private higher education institutions through acts of a legislative body, except grants and contracts. These funds are for meeting current operating expenses, not for specific programs or projects. Also included are state need-based awards to undergraduates in private institutions. These monies are divided by FTE enrollment in private institutions to arrive at the private appropriation per-student variable.

Outside Monies equals the sum of federal government appropriations, federal grants and contracts, state grants and contracts, private gifts, grants, and contracts, and endowment income. This measure includes funds for specific research projects or programs, public service, etc. The federal grants and contract figures include pell grant revenues on the survey form but have been excluded in the finance data tape and are therefore not included in this figure. This figure is divided by the public FTE enrollment.

Percentage of the Population College-aged equals the $\mathbf{1 8 - 2 4}$ years old population in a state divided by the total state population in 1988.

Educational Attainment equals the median number of school years completed in a state according to the 1980 Census.

Per-Capita Income equals 1988 per-capita income per state.

Population equals 1988 state population.
Degree of Urbanization equals percent of a state's population living in metropolitan areas in 1988.

Population Density equals population per square mile in a state in 1988.
Expected Increase in After-tax Future Earnings equals the difference between the 1980 weighted $\log$ mean earnings of male and female college graduates, ages $25-34$, working 35 or more hours per week and the 1980 weighted $\log$ mean earnings of male and female high school graduates, ages $\mathbf{2 5 - 3 4}$, working 35 or more hours per week in a state.

Foregone Earnings equals the weighted average of the $1980 \log$ mean earnings of male and female high school graduates, ages 18-24, working 35 or more hours per week in a state.

Marginal Tax Rate is approximated by the average tax rate in a state. The estimated tax rate in a state in 1988 is equal to the 1988 Total Own Source Tax Revenue (including property, sales, individual income, corporate income and other tax revenue) divided by 1988 Gross State Product. These figures were gathered from the publication State Government Finances in 1988 and the December 1991 Survey of Current Business.

Net Migration equals the percent of college graduates that remain in a state net of migration. This is approximated by the percent of college educated in state j divided by the percent of college educated in the U.S. in the numerator and the college participation rate in state j divided by the college participation rate in the U.S. in the denominator.

Percent Junior College Enrollment equals the percent of FTE junior college students in a state enrolled at public institutions.

## CHAFTER 5. ONE-SECTOR MODEL EMPIRICAL RESULTS

## Variable Definitions

In this section, the regression equations implied by the theory in Chapter 3 are developed and tested using linear regression techniques. The public per-student subsidy is regressed on the independent variables suggested by the theory. The variables and their abbreviations are described in Table 1.

Table 1. Variable Definitions.

| SUBSTD | $=$ state appropriations and need-based aid for public institutions and public students per FTE public student (dollars). |
| :---: | :---: |
| PERPOP18 | $=$ percentage of college aged people, 18-24 years old, in a state in 1988. |
| SCHOOL | $=$ median school years completed in a state in 1988. |
| PERCAPIN | $=1988$ per capita income in a state (dollars). |
| POPLN | 1988 state population (thousands). |
| URBAN | $=$ percentage of a state's citizens living in urban areas in 1988. |
| POPSQMI | population per square mile in a state in 1988. |
| LNFUTERN | $=$ the differential between log mean earnings of college graduates, ages 25-34, working 35 or more hours per week and log mean earnings of high school graduates, ages $25-34$, working 35 or more hours per week in a state. |
| LNFOREGO | $=$ the average of log mean earnings of high school graduates, ages 18-24, working 35 or more hours per week in a state. |
| AVETAXRT | $=$ the 1988 average tax rate of a state. |
| NETMIG | $=$ index of the percentage of college graduates that remain in a state. |
| OUTSTD | $=$ outside funding per FTE student in 1988 (dollars). |
| PVORIENT | $=$ the percentage of private school students in a state in 1988. |
| PVAPSTD | $=$ state appropriations and need-based awards per FTE student in private schools in 1988. |
| JUCOENRL | $=$ the percentage of public students in a state enrolled in junior colleges in 1988. |

## Regression Procedure

The one-sector model, using the cross-sectional data described previously, was run using SAS (Statistical Analysis Software). Regression equations were run for the three dependent variables, per-student subsidy for public students, average public tuition per student, and public enrollment. All of the estimating equations have the same right-hand side variables so the ordinary least squares procedure was applied equation-by-equation without loss of efficiency or introduction of bias.

The variable OUTSTD, which is comprised of competitive funding sources per student other than state appropriations, includes federal appropriations to institutions. Federal appropriations were included in this variable even though it is conceivable that these monies have a separate effect on the dependent variable, state appropriations to higher education. Regressions were run with federal appropriations as a separate explanatory variable as well as with the restriction that the parameter estimates for outside funding and federal appropriations were the same. In the regression where federal appropriations was a separate explanatory variable, the parameter estimate was not significantly different from zero. In the restricted model that tested the hypothesis that the parameter estimates for federal appropriations and outside funding sources were the same, the Ftest indicated that the hypothesis could not be rejected. Federal appropriations were considered an important variable due to previous studies, so, they were included in the variable OUTSTD.

Partial residual plots were run to test for linearity between the dependent and independent variables. The plots showed some indication of outlying variables that may unduly influence the parameter estimates so regressions were run excluding Alaska and Hawaii to test the impact that these observations have on the parameter estimates.

Variance Inflation Factors (VIF) were computed to search for evidence of multicollinearity. Values of the VIF statistic greater than 1.0 indicate that some degree of multicollinearity exists. The criteria or rule of thumb provided by Judge, et.al. is that a VIF statistic greater than 5.0 indicates severe multicollinearity (Judge, et.al., 1988). A consequence of multicollinearity is that the variances
of the parameters are large, so the estimate may be imprecise. In this study, evidence of multicollinearity was found using the VIF statistic and therefore, caution must be exercised when interpreting the parameter estimates. The parameter estimates do however, provide insight about the influence of the explanatory variables on the dependent variables.

Plots of the residuals versus the predicted values of the dependent variables were done to test for departures from linearity and non-constant variance. All of the per-student subsidy regression plots were within accepted guidelines.

## Regression Equation

The one-sector model can be expressed with the following regression equation.

```
SUBSTD \(=\beta_{0}+\beta_{1}\) PERPOP \(18+\beta_{2} S C H O O L+\beta_{3}\) PERCAPIN \(+\beta_{4} P O P L N+\beta_{5}\) URBAN
    \(+\beta_{6}\) POPSOMI \(+\beta_{7}\) LNFUTERN \(+\beta_{8}\) LNFOREGO \(+\beta_{9}\) AVETAXRT \(+\beta_{10}\) NETMIG
    \(+\beta_{11}\) OUTSTD \(+\beta_{12}\) PVORIENT \(+\beta_{13}\) PVAPSTD \(+\beta_{14}\) JUCOENRL \(+E\)
```

The regression results for the 50 states are presented in Table 2. Parameter estimates, $\mathbf{t}$ ratios, $\mathbf{R}^{\mathbf{2}}$, adjusted $\mathbf{R}^{\mathbf{2}}$, and VIF statistics are stated. Regression equations and results for public tuition and public enrollment are found in Table 1A and Table 2A in Appendix A.

## Empirical Findings vs. Theoretical Predictions for the 50 States

The regression results listed in Table 2 provide some interesting insights into the factors that influence per-student state appropriations for higher education. The theory outlined in Chapter 3 explicitly predicted the direction of the effect on the subsidy variable of changes in seven of the fourteen variables considered relevant. The regression results are consistent with the theory predictions of the effects on the subsidy variable for three of the seven variables; however, of the four that are inconsistent with theory predictions, two are not significant at the five and ten percent confidence level.

Table 2. Regression Results for One-Sector Model of Per-Student Subsidies in the $\mathbf{5 0}$ States.

| Variable | Parameter Estimate | T Values | VF | $\mathbf{R}^{2}=.6079$ <br> adj. $\mathbf{R}_{2}=.4511$ |
| :--- | ---: | :---: | :---: | :---: |
|  |  |  |  |  |
| INTERCEP | -32193 | -1.071 | 0.00 |  |
| PERPOP18 | 434.77 | 1.132 | 1.64 |  |
| SCHOOL | -345.30 | $-1.840^{* *}$ | 5.68 |  |
| PERCAPIN | 0.18 | 1.622 | 4.21 |  |
| POPLN | 49.93 | 1.149 | 2.40 |  |
| URBAN | -39.52 | $-2.992^{*}$ | 3.81 |  |
| POPSOMI | 1.39 | 1.109 | 3.82 |  |
| LNFUTERN | 123.45 | $3.459^{*}$ | 2.44 |  |
| LNFOREGO | 72.09 | $3.381^{*}$ | 2.02 |  |
| AVETAXRT | 485.84 | $2.723^{*}$ | 1.83 |  |
| NETMIG | 12.10 | $1.818^{* *}$ | 2.30 |  |
| OUTSTD | 0.68 | $1.934^{* *}$ | 2.18 |  |
| PVORIENT | 60.60 | 0.035 | 2.32 |  |
| PVAPSTD | -1.45 | $-2.195^{*}$ | 2.28 |  |
| JUCOENRL | 14.07 | 0.870 | 2.13 |  |

[^0]Educational attainment, percentage of population living in urban areas, and per-student state appropriations for private schools were found to negatively influence per-student state appropriations for public schools. The expected increase in future earnings, foregone earnings, average tax rate, migration index, and outside funding per-student variables positively influence the per-student state appropriation for public higher education.

Urbanization was predicted in the theory to positively affect per-student state appropriations for public schools due to the increased valuation of the external benefits from higher education as well as the increased costs associated with higher education provision in urban areas. The negative sign of the urbanization variable is consistent with previous findings and could be due to the increased state budgetary pressures associated with high urban populations in a state. The public tuition and enrollment regressions do not provide any insight about what has occurred in the market since the urbanization parameter estimates are not significant in either case.

The one-sector model predicts that outside funding per student would negatively affect the perstudent subsidy but the sign was positive at the ten percent significance level. This result supports Bowen's "revenue theory of costs" according to which institutions spend increased revenues on new projects and therefore, costs per student may actually rise with an increase in funds. The public tuition regression results support this theory as well since the parameter estimate for outside funding is positive and significant. Outside funding could also be an indicator of quality; therefore, the higher faculty costs associated with prestigious colleges and universities could be responsible for the positive sign of the outside funding parameter estimate.

The effect of an increase or difference in average educational attainment on the optimal perstudent subsidy was ambiguous in the theory due to the conflicting effects of shifts in the private demand curve for public higher education, $D_{E}$, and the public demand curve for higher education, PVEB. The negative parameter estimate for median school years completed indicates that the rightward shift in $D_{E}$ outweighs the rightward shift in PVEB. The greater influence of the shift in $D_{E}$ could indicate a preference by educated citizens to privately fund higher education and rely less on state appropriations. The parameter estimate for average educational attainment is positive as predicted in the theory and significant for the public enrollment regression but insignificant in the public tuition regression.

The positive signs of the parameter estimates for future earnings, average tax rate and migration index indicate that if states expect to increase their revenue from the increased earnings of college graduates, increased tax rates and increased percentage of college graduates that remain in the state, then it is in their fiscal best interest to increase funding for higher education. This supports the Holcombes' idea of government having an equity interest in funding education (Holcombe and Holcombe, 1984). The premise of their 1984 article was that the federal government's investment in higher education is a rational expenditure due to the income tax system. The income tax system grants the federal government a portion of the lifetime earnings of individuals so the government can be viewed as owning a "share" of the individual's increased Lifetime wealth. The regression results of this study support the idea of government acting as shareholders in individual's human capital investment at the state level. The positive parameter estimates for future earnings, average tax rate, and the migration index indicate that state governments and the citizens that they represent are influenced by the pecuniary aspects of higher education investment.

The public tuition and enrollment regression estimates for the future earnings, average tax rate and migration variables are listed in Appendix A. The future earnings parameter estimate in the public tuition regression supports the conclusion that the upward shift in the PVEB curve outweighs the upward shift in the private demand curve, $\mathrm{D}_{\mathrm{E}}$. The public enrollment regression results are inconclusive since the parameter estimate for the future earnings variable is not significant.

The average tax rate coefficient is negative and significant in the public tuition regression and insignificant in the public enrollment model. The negative coefficient in the tuition regression is as predicted by the theory but it is unclear whether this is due to the rightward shift in PVEB or the leftward shift in $D_{E}$ since the results for the enrollment variable are insignificant.

The migration, or the percentage of college graduates that remain in a state, parameter estimates in the public tuition and enrollment regressions are significant at the ten percent
confidence level and somewhat puzzling. The parameter estimate for the public tuition regression is negative as predicted by the theory but the parameter estimate in the enrollment regression is the opposite of the theoretical prediction and is negative. This seems to imply that a leftward shift in the private demand curve for public higher education, $D_{E}$, has occurred.

The foregone earnings variable was found to positively affect the optimal per-student subsidy level. This indicates that the leftward shift of the private demand curve for public education was larger than the leftward shift in the external benefits curve. The negative parameter estimate for foregone earnings in the public tuition regression support this conclusion. It appears from the regression results then that the state's valuation of the external benefits from higher education is less responsive to foregone income than the private demand curve for public education. The states may be less responsive since the taxes on foregone earnings is a small fraction of the earnings lost to the individual.

State appropriations for private institutions and for private students negatively affect the optimal per-student public subsidy as predicted in the theory. This is because the subsidy level is dependent upon total public and private enrollment, not just public enrollment, and as discussed earlier, total enrollment rises. Thus, the negative parameter estimate indicates that public institutions compete with private institutions for funding, and in this case, lose more per student than the private institutions gain per student.

The regression results for public tuition and public enrollment can be found in Appendix 1. The results for the public tuition model follows the theory fairly well as seen by Table 1A. Two of the parameter coefficients do not have the predicted sign, but, only one, OUTSTD, was significantly different from zero.

The public enrollment regression results are listed in Table 2A of Appendix 1. The model has a very large $R^{\mathbf{2}}$ due to the population variable. Four of the signs of the parameters are different
from theory predictions. Only one of these four, NETMIG, is significantly different from zero at the five and ten percent significance levels.

## Empirical Results for the 48 Contiguous States

Table 3 provides the regression results for the 48 contiguous states. This model was run because Alaska and Hawaii were often outliers in the explanatory and dependent variables. The regression equations and results for the public tuition and enrollment regressions are included in Appendix A, Tables 3A and 4A respectively.

The model described above does not have as much explanatory power with Alaska and Hawaii deleted from the sample. The $\mathbf{R}^{\mathbf{2}}$ and adjusted $\mathbf{R}^{\mathbf{2}}$ statistics are lower, but, there are a greater number of variables that are significant at the five percent level. In this regression, population density and the percentage of public junior college enrollment are now significant while average tax rate and state appropriations for private students are not. Several of the parameter estimates are greatly influenced by the deletion of these observations. This may be due in part to multicollinearity or the fact that as outliers, Alaska and Hawaii were high-leverage, high-impact observations.

Population density is found to positively affect the optimal per-student subsidy. This supports the "good neighbor" argument for funding higher education since it was posited that one of the effects of an increase in population density would be a rightward shift in the PVEB curve due to the increased valuation of having educated neighbors. The regression results for public tuition and enrollment are inconclusive.

The percentage of public students that attend junior colleges has a positive effect on the optimal per-student subsidy as well. The theory predicted that junior college enrollment decreases the cost of educating public students and therefore, the optimal per-student subsidy for public students would decline. The parameter estimate for junior college enrollment in the public tuition regression is negative and significant as predicted in theory. The positive coefficient in the subsidy

Table 3. Regression Results for One-Sector Model of Per-Student Subsidies in the 48 Contiguous States.

| Variable | Parameter Estimate | T Values | VIF | $\mathbf{R}^{2}=.5350$ <br> adj. $\mathbf{R}_{\mathbf{2}}=\mathbf{3 3 7 8}$ |
| :--- | ---: | :---: | :---: | :---: |
|  |  |  |  |  |
| INTERCEP | 5151.54 | 0.211 | 0.00 |  |
| PERPOP18 | -248.62 | -0.838 | 1.70 |  |
| SCHOOL | -405.92 | $-2.940^{*}$ | 5.51 |  |
| PERCAPIN | 0.08 | 0.987 | 4.54 |  |
| POPLN | 51.35 | 1.632 | 2.42 |  |
| URBAN | -34.24 | $-3.168^{*}$ | 4.89 |  |
| POPSQMI | 1.86 | $2.064^{*}$ | 3.83 |  |
| LNFUTERN | 76.30 | $2.457^{*}$ | 3.63 |  |
| LNFOREGO | 51.90 | $2.438^{*}$ | 3.24 |  |
| AVETAXRT | 138.39 | 0.978 | 2.03 |  |
| NETMIG | 13.86 | $2.716^{*}$ | 2.10 |  |
| OUTSTD | 0.64 | $2.461^{*}$ | 2.24 |  |
| PVORIENT | 809.77 | 0.659 | 2.29 |  |
| PVAPSTD | -0.60 | -1.198 | 2.52 |  |
| JUCOENRL | 22.17 | $1.845^{* *}$ | 2.05 |  |

[^1]regression may be due to an increased commitment by a state to promote higher education by using junior colleges to provide easier access for non-traditional and marginal students.

## Conclusions

The empirical results for the one-sector model provide some interesting clues about the effects of changes in the exogenous variables on the optimal state per-student subsidy in the public higher education market. The most significant finding is that the variables that heavily influence future state government income, such as increased future earnings, average tax rates and the net migration index, are significant in one or both of the regression models described above while the "good neighbor" justification for funding higher education is not as influential, except in the case of population density in the 48 -state regression results.

Variables that were expected to provide cost savings to states, such as outside funding and junior college enrollment, and therefore decrease the optimal state per-student subsidy, were found to have the opposite effect indicating that there must be other influences not predicted by the onesector version of the theory.

The regression results for the 50 states contradicted the signs of the parameters outlined in the theory for two variables at the five and ten percent confidence level while the results for the 48 contiguous states were inconsistent three times with the theory. This prompted the development of the two-sector model outlined in the next chapter to see if a more explicit treatment of the private sector changes the theory predictions and if the empirical results are more consistent with a twosector model.

## CHAPTER 6. TWO-SECTOR MODEL OF HIGHER EDUCATION

## Two-Sector Theory

The theory in this section recognizes the interrelations of the private and public higher education markets and accommodates these relationships by including in the two demand equations terms for the price in the other sector.

The higher education market in a state can be depicted with the graphs in Figure 15. As


Total Enrollment



Figure 15. The Higher Education Market with the External Benefits Curve
previously discussed in the private enrollment section of Chapter 3, the PVEB curve is a function of total enrollments, not public enrollment solely. Thus, as total higher education enrollment in a state rises, there is a lower valuation for each additional student. The PVEB curve has an intercept term of $\left(A-a Q_{V}\right)$ in the public market diagram reflecting the fact that the valuation of the benefits


Figure 16: Equilibrium in the Two-Sector Higher Education Market
from higher education is dependent upon total enrollment, not public enrollment alone. The three graphs in Figure 15 can be combined into one graph as in Figure 16.

Figure 16 depicts the relationship between the two higher education markets due to the external benefits curve, PVEB. The supply and demand curves for the private higher education market are represented as $S_{V}$ and $D_{V}$ respectively. The private education market has an equilibrium enrollment of $\mathrm{Q}_{v}{ }^{\circ}$ and tuition of $\mathrm{P}_{\mathrm{v}}{ }^{\circ}$.

The public higher education market has an origin at $\mathrm{Q}_{\mathrm{v}}{ }^{\text {i }}$ in this diagram. Hence, the public higher education demand and supply curves, $D_{E}$ and $S_{E}$ have intercept terms of (B) and (C) at this point. The intercept term of the PVEB curve in the public higher education market is at ( $\mathrm{A}-\mathrm{aO}_{\mathrm{V}}{ }^{\circ}$ ) because the valuation of the external benefits of higher education is determined by total enrollment.

Thus, the intercept of the PVEB curve in the public higher education market is higher with lower private enrollment and vice versa.

Equilibrium in the public higher education is seen by the intersection of $D_{s}$ and $S_{B}$, the total demand curve for public higher education and the supply curve for public higher education. $D_{s}$ is the vertical summation of the external benefits curve PVEB and the private demand curve for public higher education $D_{E}$ and has an intercept of $\left[\left(A-a Q_{V}{ }^{\circ}\right)+B\right]$. The equilibrium public enrollment, tuition, and per-student subsidy are $\mathrm{Q}_{\mathrm{E}}{ }^{*}, \mathrm{P}_{\mathrm{E}}{ }^{*}$, and $\mathrm{S}^{*}$ respectively.

The preceding graph is useful in showing the relationship between the two markets due to the external benefits curve; however, separating the two markets for higher education is helpful when considering shifts in the market curves. The two markets are graphically represented in Figure 17.

The demand, supply and external benefit curves for the two markets can be represented by the following equations. In this section of the theory, the demand curves for both the private and public sectors include the others' price terms to reflect the influence of the two markets on each other.



Flgure 17: Equilibrium in the Private and Public Higher Education Markets

## Private Market

(66) $D_{v}: P_{v}{ }^{D}=W-w Q_{V}+k P_{E}{ }^{D}$
(68) $S_{v}: P_{v}{ }^{s}=U+u Q_{V}$
(71) Equilibrium: $\mathbf{P}_{\mathbf{v}}{ }^{\mathbf{D}}=\mathbf{P}_{\mathbf{v}}{ }^{\mathbf{s}}$

## Public Market

(67) $\quad D_{E}: P_{B}{ }^{D}=B-b Q_{E}+P_{V}{ }^{D}$
(69) $\quad S_{B}: P_{B}^{s}=C+c Q_{B}$
(70) PVEB: $S=A-a Q_{V}-\mathrm{aQ}_{\mathrm{B}}$
(72) Equilibrium: $\mathrm{P}_{\mathrm{E}}{ }^{\mathrm{D}}+\mathrm{S}=\mathrm{P}_{\mathrm{E}}{ }^{\mathrm{S}}$
$Q_{V}$ and $Q_{B}$ are private and public enrollments respectively, $P_{V}$ and $P_{B}$ are private and public tuition respectively, and $S$ is the optimal per-student subsidy to public students. $W, U, A, B, C, w, u, a, b$, $c, k, 1$ are positive parameters. The intercept terms for both markets are linear functions of the exogenous variables and can be represented in the following manner.

$$
\begin{array}{ll}
A=\sum_{i=1}^{n} \alpha_{i} X_{i} & B=\sum_{i=1}^{n} \gamma_{i} X_{i} \quad C=\sum_{i=1}^{n} \theta_{i} X_{i} \\
U=\sum_{i=1}^{n} \lambda_{i} X_{i} & W=\sum_{i=1}^{n} \delta_{i} X_{i}
\end{array}
$$

The optimal solutions can be found by using equations (66) through (72). The optimal solutions are:

$$
\begin{align*}
S^{*}=A & -a\left\{\frac{[(w+u)-l u k] A+[(w+u)-a k] B+[-(w+u)+l u k] C+(l u-a) W+[(l w+a)-a l k] U}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right\}  \tag{73}\\
& -a\left\{\frac{W-U}{w+u}+\frac{k}{w+u}\left[\frac{-b(w+u) A+(a+c)(w+u) B+b(w+u) C+[l u(a+c)+a b] W+[l w(a+c)-a b] U}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right]\right\}
\end{align*}
$$

$$
\begin{align*}
Q_{E}^{*} & =\frac{[(w+u)-l u k] A+[(w+u)-a k] B+[-(w+u)+l u k] C}{(a+b+c)(w+u)-l u k(a+c)-a b k}  \tag{74}\\
& +\frac{(l u-a) W+[(l w+a)-a l k] U}{(a+b+c)(w+u)-l u k(a+c)-a b k}
\end{align*}
$$

$$
\begin{equation*}
P_{E}^{:}=\frac{-b(w+u) A+(a+c)(w+u) B+b(w+u) C+[l u(a+c)+a b] W+[l w(a+c)-a b] U}{(a+b+c)(w+u)-l u k(a+c)-a b k} \tag{75}
\end{equation*}
$$

$$
Q_{\dot{v}}^{\bullet}=\frac{W-U}{W+u}
$$

$$
+\frac{k}{w+u}\left[\frac{-b(w+u) A+(a+c)(w+u) B+b(w+u) C+[\operatorname{lu}(a+c)+a b] W+[\operatorname{lw}(a+c)-a b] U}{(a+b+c)(w+u)-\operatorname{luk}(a+c)-a b k}\right]
$$

$$
\begin{equation*}
P_{v}^{*}=\frac{u}{w+u} W+\frac{w}{w+u} U \tag{77}
\end{equation*}
$$

$$
+\frac{u k}{w+u}\left[\frac{-b(w+u) A+(a+c)(w+u) B+b(w+u) C+[l u(a+c)+a b] W+[l w(a+c)-a b] U}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right]
$$

As in Chapter 3, comparative static analysis can be used to determine the influence of the explanatory variables on the five dependent variables: subsidy, public tuition, public enrollment, private tuition, and private enrollment. The exogenous variables remain the same except that private orientation is not included as an independent variable because private enrollments are now determined within the model.

The partial derivatives with respect to the intercept terms for the dependent variables follow. Equations $78-82$ list the partial derivatives with respect to the intercept terms for the subsidy variable.

$$
\begin{equation*}
\frac{\partial S^{*}}{\partial A}=1-a\left[\frac{(w+u)-l u k+a k b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right] \frac{2}{<} 0 \tag{78}
\end{equation*}
$$

$$
\begin{align*}
& \frac{\partial S^{*}}{\partial B}=\frac{-a(w+u)-a k c}{(a+b+c)(w+u)-l u k(a+c)-a b k} \geq 0  \tag{79}\\
& \frac{\partial S^{*}}{\partial C}=\frac{a(w+u)-a l u k-a k b}{(a+b+c)(w+u)-l u k(a+c)-a b k} \geq 0 \tag{80}
\end{align*}
$$

(81)

$$
\frac{\partial S^{*}}{\partial W}=\frac{-a(l u-a)}{(a+b+c)(w+u)-l u k(a+c)-a b k}-\frac{a}{w+u}
$$

$$
-\frac{a k}{w+u}\left\{\frac{l u(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right\} \geq 0
$$

$$
\begin{equation*}
\frac{\partial S^{*}}{\partial U}=\frac{-a[(l w+a)-a l k]}{(a+b+c)(w+u)-l u k(a+c)-a b k}+\frac{a}{w+u}-\frac{a k}{w+u}\left\{\frac{l w(a+c)-a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right\} \geq 0 \tag{82}
\end{equation*}
$$

Equations 83-87 are the partial derivatives with respect to the intercept terms for the public tuition variable.

$$
\begin{equation*}
\frac{\partial P_{E}^{*}}{\partial A}=\frac{-b(w+u)}{(a+b+c)(w+u)-l u k(a+c)-a b k} \geq 0 \tag{83}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial P_{E}^{*}}{\partial B}=\frac{(a+c)(w+u)}{(a+b+c)(w+u)-l u k(a+c)-a b k} \geq 0 \tag{84}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial P_{E}^{*}}{\partial C}=\frac{b(w+u)}{(a+b+c)(w+u)-l u k(a+c)-a b k} \geq 0 \tag{85}
\end{equation*}
$$

$$
\begin{align*}
& \frac{\partial P_{E}^{*}}{\partial W}=\frac{l u(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a b k} \geq 0  \tag{86}\\
& \frac{\partial P_{E}^{*}}{\partial U}=\frac{\operatorname{lW}(a+c)-a b}{(a+b+c)(w+u)-l u k(a+c)-a b k} \geq 0 \tag{87}
\end{align*}
$$

The partial derivatives for the public enrollment variable with respect to the intercept terms are listed in equations 88-92.

$$
\begin{equation*}
\frac{\partial Q_{E}^{*}}{\partial A}=\frac{(w+u)-u k k}{(a+b+c)(w+u)-\operatorname{luk}(a+c)-a b k} \geq 0 \tag{88}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial Q_{E}^{*}}{\partial B}=\frac{(w+u)-a k}{(a+b+c)(w+u)-\operatorname{lu} k(a+c)-a b k} \geq 0 \tag{89}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial Q_{\dot{E}}}{\partial C}=\frac{-(w+u)+l u k}{(a+b+c)(w+u)-l u k(a+c)-a b k} \geq 0 \tag{90}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial Q_{E}^{*}}{\partial W}=\frac{u u-a}{(a+b+c)(w+u)-l u k(a+c)-a b k} \geq 0 \tag{91}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial Q_{E}^{*}}{\partial U}=\frac{(l w+a)-a l k}{(a+b+c)(w+u)-l u k(a+c)-a b k} \geq 0 \tag{92}
\end{equation*}
$$

Equations 93-97 are the partial derivatives of private tuition with respect to the intercept terms.

$$
\begin{equation*}
\frac{\partial P_{V}^{*}}{\partial A}=\frac{-u k b}{(a+b+c)(w+u)-l u k(a+c)-a b k} \geq 0 \tag{93}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial P_{V}^{*}}{\partial C}=\frac{u k b}{(a+b+c)(w+u)-l u k(a+c)-a b k} \geq 0 \tag{95}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial P_{v}^{*}}{\partial W}=\frac{u}{w+u}+\frac{u k}{w+u}\left[\frac{u u(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right] \frac{\geq}{<} 0 \tag{96}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial P_{V}^{*}}{\partial U}=\frac{w}{w+u}+\frac{u k}{w+u}\left[\frac{l w(a+c)-a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right] \geq 0 \tag{97}
\end{equation*}
$$

The partial derivatives of the private enrollment variable with respect to the intercept terms are provided in equations 98-102.

$$
\begin{align*}
& \frac{\partial Q_{V}^{*}}{\partial W}=\frac{1}{w+u}+\frac{k}{w+u}\left[\frac{u(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right] \geq 0  \tag{101}\\
& \frac{\partial Q_{V}^{*}}{\partial U}=-\frac{1}{w+u}+\frac{k}{w+u}\left[\frac{l w(a+c)-a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right] \geq 0
\end{align*}
$$

The signs of the partial derivative terms of the dependent variables with respect to the intercept terms cannot be determined without the restrictions that l and k are less than 1.0 and that $\mathrm{w}+\mathrm{u}>\mathrm{a}$. The first restriction is reasonable because it is unlikely that the respective demand curves would shift as much as the change in the price of the other good. In other words, it is unlikely that enrollment in the public sector is as responsive to a change in the price of the private sector as it is to a change in its' own price and vice versa. The second restriction, $w+u>a$, requires that the sum of the slopes of the demand and supply curves in the private higher education market be greater than the slope of the PVEB curve. These assumptions allow for the signs of some of the partial derivatives with respect to the intercept terms to be determined since the denominator is positive. The signs of the intercept partials are as follows:

$$
\begin{equation*}
\frac{\partial P_{E}^{*}}{\partial A}<0 \frac{\partial P_{E}^{*}}{\partial B}>0 \frac{\partial P_{E}^{*}}{\partial C}>0 \frac{\partial P_{E}^{*}}{\partial W}>0 \frac{\partial P_{E}^{*}}{\partial U} \frac{\geq 0}{<} \tag{103}
\end{equation*}
$$

$$
\begin{align*}
& \frac{\partial Q_{E}^{*}}{\partial A}>0 \frac{\partial Q_{E}^{*}}{\partial B}>0 \quad \frac{\partial Q_{E}^{*}}{\partial C}<0 \quad \frac{\partial Q_{E}^{*}}{\partial W} \geq 0 \quad \frac{\partial Q_{E}^{*}}{\partial U}>0  \tag{104}\\
& \frac{\partial S^{*}}{\partial A} \geq 0 \quad \frac{\partial S^{*}}{\partial B}<0 \quad \frac{\partial S^{*}}{\partial C} \geq 0 \quad \frac{\partial S^{*}}{\partial W} \geq 0 \quad \frac{\partial S^{*}}{\partial U} \geq 0  \tag{105}\\
& \frac{\partial Q_{V}^{*}}{\partial A}<0 \quad \frac{\partial Q_{V}^{*}}{\partial B}>0 \frac{\partial Q_{V}^{*}}{\partial C}>0 \quad \frac{\partial Q_{V}^{*}}{\partial W}>0 \quad \frac{\partial Q_{V}^{*}}{\partial U} \frac{\geq}{<}<0  \tag{106}\\
& \frac{\partial P_{V}^{*}}{\partial A}<0 \quad \frac{\partial P_{V}^{*}}{\partial B}>0 \quad \frac{\partial P_{v}^{*}}{\partial C}>0 \quad \frac{\partial P_{V}^{*}}{\partial W}>0 \quad \frac{\partial P_{V}^{*}}{\partial U} \geq 0 \tag{107}
\end{align*}
$$

## Two-Sector Comparative Statics

## Percent of the Population College-aged

The percentage of the population college-aged in a state would initially affect the private demand curves for both public and private education. An increase in the percentage of the population college-aged results in a rightward shift of the private demand curve for private education as shown in Figure 18. Private enrollments and private tuition would both tend to rise toward $\mathrm{Q}_{\mathrm{v} 1}$ and $\mathrm{P}_{\mathrm{v} 1}$ respectively.

The private demand curve for public education would also rise from an increase in the percentage of the population college-aged. A rightward shift in $\mathrm{D}_{\mathrm{E}}$ would result in a rightward shift of the total demand curve for public higher education, $D_{s}$, and therefore public tuition, and public enrollments would tend to rise and the subsidy would tend to fall toward $P_{E 1}, Q_{E 1}$, and $S_{1}$.

There are several complicating effects before a new equilibrium is attained. The external benefits curve, PVEB, graphed here relative to public enrollment, is also dependent upon private enrollment levels and would shift leftward in Figure 18 due to the increase in private enrollment. The total demand curve for public education would therefore shift leftward and tend towards an


Figure 18. The Initial Effects of an Increase in the Percentage of the Population College-aged on Higher Education Equilibrium
equilibrium with lower public enrollment, higher public tuition, and lower subsidy per public student. The private higher education market would subsequently be affected due to the decrease in the public tuition. The private demand curve for private higher education would shift leftward due to the decrease in public tuition. These types of secondary effects will continue until an equilibrium is established. Graphing the complete adjustment process is hopelessly complex. Mathematics provides a better route to the new equilibrium.

The final effects of the shifts in both private demand curves for public and private higher education can be demonstrated by the following comparative static equations. Let the percentage of the population college-aged be represented by $X_{1}$. Then, using equations (67), (69), (70), (73), and (78) through (82) the effect of a change in the percentage of the population college-aged on the optimal per-student subsidy for public students can be seen.

$$
\frac{\partial S^{*}}{\partial X_{1}}=\frac{\gamma_{1}[-a(w+u)-a k c]}{(a+b+c)(w+u)-l u k(a+c)-a b k}+\delta_{1}\left[\frac{-a(l u-a)}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right]
$$

$$
\begin{align*}
& +\delta_{1}\left[\frac{-a}{w+u}-\frac{a k}{w+u}\left\{\frac{u(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right\}\right] \frac{\geq}{<} 0  \tag{108}\\
& \text { because } \frac{\partial S^{*}}{\partial B} \frac{\geq}{<} 0 \frac{\partial S^{*}}{\partial W} \frac{>}{<} 0 \text { and } \gamma_{1}>0, \delta_{1}>0
\end{align*}
$$

As demonstrated by equation (108), an increase in the percentage of the population college-aged has an ambiguous effect on the optimal per-student subsidy.

Equations (67), (69), (70), (75), and (83) through (87) can be used to determine the effect of an increase in the percentage of the population college-aged on public tuition levels.

$$
\begin{align*}
\frac{\partial P_{E}^{*}}{\partial X_{1}} & =\frac{\gamma_{1}(a+c)(w+u)+\delta_{1}[l u(a+c)+a b]}{(a+b+c)(w+u)-l u k(a+c)-a b k}>0  \tag{109}\\
& \text { because } \frac{\partial P_{E}^{*}}{\partial B}>0 \frac{\partial P_{E}^{*}}{\partial W}>0 \text { and } \gamma_{1}>0, \delta_{1}>0
\end{align*}
$$

Equation (109) shows the positive effect of an increase in the percentage of the population collegeaged on public tuition levels.

The ambiguous effect of an increase in the population college-aged on public enrollments can be demonstrated using equations (70), (72), (73), (74), and (88) through (92).

$$
\begin{equation*}
\frac{\partial Q_{E}^{*}}{\partial X_{1}}=\frac{\gamma_{1}[(w+u)-a k]+\delta_{1}(\mu-a)}{(a+b+c)(w+u)-l u k(a+c)-a b k} \geq 0 \text { because } \frac{\partial Q_{E}^{*}}{\partial W} \geq 0 \tag{110}
\end{equation*}
$$

Equations (66), (68), (71), (76), (77), and (93) through (102) can be used to express the effect of an increase in the percentage of the population college-aged on private enrollments and private tuition.

$$
\begin{gather*}
\frac{\partial Q_{V}^{*}}{\partial X_{1}}=\frac{\gamma_{1} k(a+c)}{(a+b+c)(w+u)-l u k(a+c)-a k b}+\delta_{1}\left[\frac{1}{w+u}+\frac{k}{w+u}\left\{\frac{u(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a k b}\right]\right]>0  \tag{111}\\
\quad \text { because } \frac{\partial Q_{V}^{*}}{\partial B}>0, \frac{\partial Q_{V}^{*}}{\partial W}>0 \quad \gamma_{1}>0 \text { and } \delta_{1}>0 \\
\frac{\partial P_{V}^{*}}{\partial X_{1}}=\frac{\gamma_{1} u k(a+c)}{(a+b+c)(w+u)-l u k(a+c)-a b k}+\delta_{1}\left[\frac{u}{w+u}+\frac{u k}{w+u}\left\{\frac{u(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right]>0\right.  \tag{112}\\
\text { because } \frac{\partial P_{V}^{*}}{\partial B}>0 \quad \frac{\partial P_{V}^{*}}{\partial W}>0, \quad \gamma_{1}>0 \text { and } \delta_{1}>0
\end{gather*}
$$

These partial derivatives are clearly positive, therefore, an increase in the percentage of the population college-aged would raise the optimal private enrollment and private tuition as expected with a rightward shift in the private demand curve for private higher education.

## Average Educational Attainment

Average educational attainment influences the private demand curves for public and private higher education as well as the external benefits curve, PVEB. As was discussed in Chapter 3, an increase in average educational attainment would shift both demand curves for higher education and the external benefits curve rightward.

Figure 19 illustrates the tendency towards increased private enrollment and private tuition, $\mathrm{Q}_{\mathrm{v} 1}$ and $\mathbf{P}_{\mathrm{v}_{1}}$, as well as increased public enrollments, public tuition and decreased per-student subsidy $\mathrm{Q}_{\mathrm{El}}, \mathrm{P}_{\mathrm{EI}}$, and $\mathrm{S}_{1}$, that would initially tend to occur from increases in the demand curves for public and private higher education. The public higher education market is also influenced by an initial rightward shift in the PVEB curve due to the increased valuation of the external benefits from higher education. The rightward shift of PVEB would tend to increase public enrollment towards $Q_{E 1}$, decrease public tuition towards $P_{E 2}$ and increase the optimal per-student subsidy towards $S_{2}$.

The initial effects of the shifts in the three curves are mitigated by the secondary shifts that occur due to the price interrelationship between the two demand curves as well as the shift in the PVEB curve resulting from the change in the level of private enrollment. Increased private enrollment would cause the PVEB curve to shift leftward in the public market; therefore, public tuition would tend to rise and public enrollment and the per-student subsidy would tend to fall. The tuition changes in each market also have feedback effects on the two demand curves. The adjustments will continue until a new equilibrium is established.


Figure 19. The Initial Effects of an Increase in Average Educational Attainment on Higher Equilibrium Education

The final effects of an increase in average educational attainment on the five higher education variables can be determined mathematically. Let average educational attainment be $X_{2}$. Then the uncertainty of the change in the tuition and enrollment variables as well as the optimal subsidy for public students can be seen with the following partial derivatives.

$$
\begin{align*}
& \frac{\partial S^{*}}{\partial X_{2}}=\alpha_{2}\left[1-a\left\{\frac{(w+u)-l u k+a k b}{(a+b+c)(w+u)-l u k(a+c)-a k b}\right\}\right]+\gamma_{2}\left[\frac{-a(w+u)-a k c}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right] \\
& +\delta_{2}\left[\frac{-a(l u-a)}{(a+b+c)(w+u)-l u k(a+c)-a k b}-\frac{a}{w+u}-\frac{a k}{w+u}\left\{\frac{u u(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right\}\right]  \tag{113}\\
& \frac{\partial S^{*}}{\partial X_{2}} \geq 0 \text { since } \frac{\partial S^{*}}{\partial B}<0, \frac{\partial S^{*}}{\partial W} \geq 0 \quad \frac{\partial S^{*}}{\partial A} \frac{\geq}{<} 0 \quad \alpha_{2}>0, \gamma_{2}>0 \text { and } \delta_{2}>0 \text {. } \\
& \frac{\partial P_{E}^{*}}{\partial X_{2}}=\frac{-\alpha_{2} b(w+u)+\gamma_{2}(a+c)(w+u)+\delta_{2}[l u(a+c)+a b]}{(a+b+c)(w+u)-l u k(a+c)-a b k} \geq 0  \tag{114}\\
& \text { since } \frac{\partial P_{E}^{*}}{\partial A}<0, \frac{\partial P_{E}^{*}}{\partial B}>0 \frac{\partial P_{E}^{*}}{\partial W}>0 \alpha_{2}>0 \gamma_{2}>0 \text { and } \delta_{2}>0 \text {. } \\
& \frac{\partial Q_{E}^{*}}{\partial X_{2}}=\frac{\alpha_{2}[(w+u)-l u k]+\gamma_{2}[w+u-a k]+\delta_{2}(l u-a)}{(a+b+c)(w+u)-l u k(a+c)-a b k} \geq 0  \tag{115}\\
& \text { since } \frac{\partial Q_{E}^{*}}{\partial A}>0 \frac{\partial Q_{E}^{*}}{\partial B}>0 \frac{\partial Q_{E}^{*}}{\partial W} \geq 0 \quad \alpha_{2}>0, \gamma_{2}>0 \text { and } \delta_{2}>0 \\
& \frac{\partial P_{v}^{*}}{\partial X_{2}}=\frac{-\alpha_{2} u k b+\gamma_{2} u k(a+c)}{(a+b+c)(w+u)-l u k(a+c)-a b k}+\delta_{2}\left[\frac{u}{w+u}+\frac{u k}{w+u}\left\{\frac{u(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right\}\right]  \tag{116}\\
& \frac{\partial P_{V}^{*}}{\partial X_{2}} \geq 0 \text { since } \frac{\partial P_{V}^{*}}{\partial A}<0, \frac{\partial P_{V}^{*}}{\partial B}>0, \frac{\partial P_{V}^{*}}{\partial W}>0, \quad \alpha_{2}>0 \quad \gamma_{2}>0 \quad \delta_{2}>0 \\
& \frac{\partial Q_{V}^{*}}{\partial X_{2}}=\frac{-\alpha_{2} k b+\gamma_{2} k(a+c)}{(a+b+c)(w+u)-l u k(a+c)-a b k}+\delta_{2}\left[\frac{1}{w+u}+\frac{k}{w+u}\left\{\frac{l u(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a k b}\right\}\right]  \tag{117}\\
& \frac{\partial Q_{V}^{*}}{\partial X_{2}} \frac{\geq}{<} 0 \text { since } \frac{\partial Q_{V}^{*}}{\partial A}<0 \frac{\partial Q_{V}^{*}}{\partial B}>0 \frac{\partial Q_{V}^{*}}{\partial W}>0 \alpha_{2}>0 \gamma_{2}>0 \text { and } \delta_{2}>0
\end{align*}
$$

## Per-capita Income

Consider the effects of a change in per-capita income. Initially, an increase in per-capita income positively influences the private demand curves for private and public higher education as
well as the respective supply curves for each market. The external benefits curve, PVEB, is also positively influenced initially by an increase in per-capita income. The effects of the initial multiple shifts in the two markets as well as the subsequent secondary movements are ambiguous for all five of the higher education variables. Let per-capita income be $\mathrm{X}_{3}$. Then the effects of an increase in per-capita income on the five higher education variables can be shown in the ensuing equations.

$$
\begin{align*}
& \frac{\partial S^{*}}{\partial X_{3}}=\alpha_{3}\left[1-a\left\{\frac{(w+u)-l u k+a k b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right\}\right]+\gamma_{3}\left[\frac{-a(w+u)-a k c}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right] \\
& +\theta_{3}\left[\frac{a(w+u)-a l u k-a k b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right] \\
& +\delta_{3}\left[\frac{-a(l u-a)}{(a+b+c)(w+u)-l u k(a+c)-a b k}-\frac{a}{w+u}-\frac{a k}{w+u}\left\{\frac{l u(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right\}\right]  \tag{118}\\
& +\lambda_{3}\left[\frac{-a[(l w+a)-a l k]}{(a+b+c)(w+u)-l u k(a+c)-a b k}+\frac{a}{w+u}-\frac{a k}{w+u}\left\{\frac{l w(a+c)-a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right\}\right] \\
& \frac{\partial S}{\partial X_{3}} \frac{\geq}{<} 0 \text { because } \frac{\partial S^{*}}{\partial A} \geq 0, \frac{\partial S^{*}}{\partial B}>0, \frac{\partial S^{*}}{\partial C}<0, \frac{\partial S^{*}}{\partial W} \geq 0 \\
& \frac{\partial S^{*}}{\partial U} \geq 0, \alpha_{3}>0, \gamma_{3}>0, \theta_{3}>0, \delta_{3}>0 \text { and } \lambda_{3}>0 \\
& \frac{\partial P_{E}^{*}}{\partial X_{3}}=\frac{-\alpha_{3} b(w+u)+\gamma_{3}(a+c)(w+u)+\theta_{3} b(w+u)+\delta_{3}[l u(a+c)+a b]+\lambda_{3}[l w(a+c)-a b]}{(a+b+c)(w+u)-l u k(a+c)-a b k} \\
& \frac{\partial P_{E}}{\partial X_{3}} \geq 0 \text { because } \frac{\partial P_{E}^{*}}{\partial A}<0 \frac{\partial P_{E}^{*}}{\partial B}>0 \frac{\partial P_{E}^{*}}{\partial C}>0 \frac{\partial P_{E}^{*}}{\partial W}>0, \frac{\partial P_{E}^{*}}{\partial U} \geq 0,  \tag{119}\\
& \alpha_{3}>0, \gamma_{3}>0, \theta_{3}>0, \delta_{3}>0 \text { and } \lambda_{3}>0 \\
& \frac{\partial Q_{E}^{*}}{\partial X_{3}}=\frac{\alpha_{3}[(w+u)-l u k]+\gamma_{3}[(w+u)-a k]+\theta_{3}[-(w+u)+l u k]+\delta_{3}[u-a]+\lambda_{3}[(l w+a)-a l k]}{[(a+b+c)(w+u)-l u k(a+c)-a b k]} \\
& \frac{\partial Q_{E}^{+}}{\partial X_{3}} \geq 0 \text { because } \frac{\partial Q_{E}^{+}}{\partial A}>0 \frac{\partial Q_{E}^{+}}{\partial B}>0 \quad \frac{\partial Q_{E}^{+}}{\partial C}<0 \frac{\partial Q_{E}^{+}}{\partial W} \geq 0 \frac{\partial Q_{E}^{+}}{\partial U}>0,  \tag{120}\\
& \alpha_{3}>0, \gamma_{3}>0, \theta_{3}>0, \delta_{3}>0 \text {, and } \lambda_{3}>0
\end{align*}
$$

$$
\begin{align*}
\frac{\partial P_{v}^{*}}{\partial X_{3}} & =\frac{-\alpha_{3} u k b+\gamma_{3} u k(a+c)+\theta_{3} u k b}{(a+b+c)(w+u)-l u k(a+c)-a b k} \\
& +\delta_{3}\left[\frac{u}{w+u}+\frac{u k}{w+u}\left\{\frac{u u(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a k b}\right\}\right] \tag{121}
\end{align*}
$$

$$
\begin{aligned}
\frac{\partial Q_{V}^{*}}{\partial X_{3}} & =\frac{-\alpha_{3} k b+\gamma_{3} k(a+c)+\theta_{3} k b}{(a+b+c)(w+u)-l u k(a+c)-a k b} \\
& +\delta_{3}\left[\frac{1}{w+u}+\frac{k}{w+u}\left\{\frac{l u(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right\}\right] \\
& +\lambda_{3}\left[-\frac{1}{w+u}+\frac{k}{w+u}\left\{\frac{l w(a+c)-a b}{(a+b+c)(w+u)-l u k(a+c)-a k b}\right\}\right] \geq 0 \\
& \text { since } \frac{\partial Q_{V}^{*}}{\partial A}<0 \quad \frac{\partial Q_{V}^{*}}{\partial B}>0 \frac{\partial Q_{V}^{*}}{\partial C}>0 \frac{\partial Q_{V}^{*}}{\partial W}>0 \quad \frac{\partial Q_{V}^{\circ}}{\partial U} \geq 0 \\
& \alpha_{3}>0 \quad \gamma_{3}>0 \quad \theta_{3}>0 \quad \delta_{3}>0 \text { and } \lambda_{3}>0
\end{aligned}
$$

## Population

A change in the population of a state has the same effect on the two higher education markets as a change in average educational attainment; initially an increase in population will positively affect the two demand curves for higher education as well as the PVEB curve. Figure 19 can be used again to illustrate the initial tendency towards increases in private enrollment, private tuition, and public enrollment and the ambiguities with respect to public tuition and the optimal per-student
subsidy. However, as with an increase in average educational attainment, the ultimate effects of an increase in population on the five higher education variables are completely ambiguous due to the secondary adjustments that occur.

Let population be denoted as $\mathbf{X}_{4}$. Then, the final effects of an increase in the population of a state on the tuition, enrollment and subsidy variables can be demonstrated mathematically in equations (123) through (127).

$$
\begin{align*}
& \frac{\partial S^{*}}{\partial X_{4}}=\alpha_{4}\left[1-a\left\{\frac{(w+u)-l u k+a k b}{(a+b+c)(w+u)-l u k(a+c)-a k b}\right\}\right]+\gamma_{4}\left[\frac{-a(w+u)-a k c}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right] \\
& +\delta_{4}\left[\frac{-a(u-a)}{(a+b+c)(w+u)-l u k(a+c)-a k b}-\frac{a}{w+u}-\frac{a k}{w+u}\left\{\frac{u(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right\}\right]  \tag{123}\\
& \frac{\partial S^{*}}{\partial X_{4}} \geq 0 \text { since } \frac{\partial S^{*}}{\partial B}<0, \frac{\partial S^{*}}{\partial W} \geq 0 \frac{\partial S^{*}}{\partial A} \geq 0 \alpha_{4}>0, \gamma_{4}>0 \text { and } \delta_{4}>0 \\
& \frac{\partial P_{\Sigma}^{*}}{\partial X_{4}}=\frac{-\alpha_{4} b(w+u)+\gamma_{4}(a+c)(w+u)+\delta_{4}[l u(a+c)+a b]}{(a+b+c)(w+u)-l u k(a+c)-a b k} \geq 0  \tag{124}\\
& \text { since } \frac{\partial P_{E}^{*}}{\partial A}<0, \frac{\partial P_{E}^{*}}{\partial B}>0 \frac{\partial P_{E}^{*}}{\partial W}>0 \alpha_{4}>0 \gamma_{4}>0 \text { and } \delta_{4}>0 \\
& \frac{\partial Q_{E}^{*}}{\partial X_{4}}=\frac{\alpha_{4}[(w+u)-u k]+\gamma_{4}[w+u-a k]+\delta_{4}(u-a)}{(a+b+c)(w+u)-l u k(a+c)-a b k} \geq 0  \tag{125}\\
& \text { since } \frac{\partial Q_{E}^{*}}{\partial A}>0 \frac{\partial Q_{E}^{*}}{\partial B}>0 \frac{\partial Q_{E}^{*}}{\partial W} \geq 0 \quad \alpha_{4}>0, \gamma_{4}>0 \text { and } \delta_{4}>0 \\
& \frac{\partial P_{V}^{*}}{\partial X_{4}}=\frac{-\alpha_{4} u k b+\gamma_{4} u k(a+c)}{(a+b+c)(w+u)-l u k(a+c)-a b k}+\delta_{4}\left[\frac{u}{w+u}+\frac{u k}{w+u}\left\{\frac{u(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right\}\right] \\
& \frac{\partial P_{V}^{*}}{\partial X_{4}} \geq 0 \text { since } \frac{\partial P_{V}^{*}}{\partial A}<0, \frac{\partial P_{V}^{*}}{\partial B}>0, \frac{\partial P_{V}^{*}}{\partial W}>0, \quad \alpha_{4}>0 \quad \gamma_{4}>0 \quad \delta_{4}>0
\end{align*}
$$

$$
\begin{align*}
\frac{\partial Q_{V}^{*}}{\partial X_{4}}= & \frac{-\alpha_{4} k b+\gamma_{4} k(a+c)}{(a+b+c)(w+u)-l u k(a+c)-a b k}+\delta_{4}\left[\frac{1}{w+u}+\frac{k}{w+u}\left\{\frac{\mu(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a k b}\right\}\right]  \tag{127}\\
& \frac{\partial Q_{V}^{*}}{\partial X_{4}} \geq 0 \text { since } \frac{\partial Q_{V}^{*}}{\partial A}<0 \frac{\partial Q_{V}^{*}}{\partial B}>0 \frac{\partial Q_{V}^{*}}{\partial W}>0 \alpha_{4}>0 \gamma_{4}>0 \text { and } \delta_{4}>0
\end{align*}
$$

## Urbanization

The degree of urbanization in a state affects the location of the two supply curves and the PVEB curve initially. Figure 20 illustrates the initial effects of the rightward shift in PVEB and the leftward shifts in $\mathrm{S}_{\mathrm{v}}$ and $\mathrm{S}_{\mathrm{E}}$.

The leftward shift in the supply curve for the private market, $\mathrm{S}_{\mathrm{v}}$, would tend to decrease private enrollment and increase private tuition towards $\mathrm{Q}_{\mathrm{V}_{1}}$ and $\mathrm{P}_{\mathrm{v}_{1}}$. The leftward shift in the public market supply curve, $S_{E}$, would initially push the equilibrium point toward $E_{1}$ with a decline in enrollment


Figure 20: The Initial Effects of an Increase in Urbanization on Higher Education Equilibrium
toward $\mathrm{Q}_{\mathrm{El}}$, an increase in tuition toward $\mathrm{P}_{\mathrm{EI}}$ and an increase in the optimal per-student subsidy toward $\mathrm{S}_{1}$.

The rightward shift in PVEB to PVEB $_{1}$ that occurs would tend to increase enrollment toward $\mathbf{Q}_{\mathrm{E} 2}$, decrease tuition toward $\mathbf{P}_{\mathrm{E} 2}$ and increase the optimal subsidy toward $\mathbf{S}_{\mathbf{2}}$.

Again, there are secondary effects that occur due to the changes in tuition in both markets as well as the change in private enrollment. The PVEB curve would shift rightward when private enrollment declines and the two private demand curves would shift due to the price changes in the other market. Thus, the final effects of an increase in urbanization on the higher education variables must be determined mathematically.

Let the degree of urbanization in a state be denoted as $\mathrm{X}_{\mathbf{5}}$. The effects of an increase in the degree of urbanization in a state on the higher education variables are presented in following equations.

$$
\begin{align*}
\frac{\partial S^{*}}{\partial X_{5}} & =\alpha_{5}\left[1-a\left\{\frac{(w+u)-l u k+a k b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right\}\right]+\theta_{5}\left[\frac{a(w+u)-a l u k-a k b}{(a+b+c)(w+u)-l u k(a+c)-a k b}\right]  \tag{128}\\
& +\lambda_{5}\left[\frac{-a[l w+a-a l k]}{(a+b+c)(w+u)-l u k(a+c)-a b k}+\frac{a}{w+u}-\frac{a k}{w+u}\left\{\frac{l w(a+c)-a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right\}\right] \frac{2}{<} 0
\end{align*}
$$

$$
\begin{equation*}
\frac{\partial Q_{E}^{*}}{\partial X_{5}}=\frac{\alpha_{5}[(w+u)-l u k]+\theta_{5}[-(w+u)+l u k]+\lambda_{5}[(l w+a)-a l k]}{(a+b+c)(w+u)-l u k(a+c)-a b k} \geq 0 \tag{130}
\end{equation*}
$$

$$
\begin{gather*}
\frac{\partial P_{v}^{*}}{\partial X_{5}}=\frac{-\alpha_{5} u k b+\theta_{5} u k b}{(a+b+c)(w+u)-l u k(a+c)-a b k}+\lambda_{5}\left[\frac{w}{w+u}+\frac{u k}{w+u}\left\{\frac{l w(a+c)-a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right\}\right] \geq 0  \tag{131}\\
\frac{\partial Q_{V}^{*}}{\partial X_{5}}=\frac{-\alpha_{5} k b+\theta_{5} k b}{(a+b+c)(w+u)-l u k(a+c)-a b k}+\lambda_{5}\left[-\frac{1}{w+u}+\frac{k}{w+u}\left\{\frac{l w(a+c)-a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right\}\right] \frac{\geq}{<} 0 \tag{132}
\end{gather*}
$$

$$
\begin{equation*}
\frac{\partial P_{E}^{*}}{\partial X_{5}}=\frac{-\alpha_{5} b(w+u)+\theta_{5} b(w+u)+\lambda_{5}[l w(a+c)-a b]}{(a+b+c)(w+u)-l u k(a+c)-a b k} \geq 0 \tag{129}
\end{equation*}
$$

The final effects of an increase in urbanization on the higher education variables are completely ambiguous as demonstrated by equations 128 through 132.

## Population Density

An increase in the population density of a state would initially shift the demand curves for both the private and public higher education markets and the PVER curve. Figure 19 can again be used to illustrate these shifts and the initial effects associated with an increase in population density.

The upward shift in the demand curve of the private higher education market to $\mathrm{D}_{\mathrm{v}_{1}}$ would tend to increase private enrollment toward $\mathrm{Q}_{\mathrm{VI}_{1}}$ and increase private tuition toward $\mathrm{P}_{\mathrm{V} 1}$. The upward shift in the public market supply curve to $\mathrm{D}_{\mathrm{El}}$ would tend to increase public enrollment toward $\mathrm{Q}_{\mathrm{El}}$, increase public tuition toward $P_{E I}$, and a decrease the optimal per-student subsidy toward $S_{1}$. The initial upward shift in the PVEB curve would positively affect public enrollment and subsidy toward $\mathrm{Q}_{\mathrm{EI}}$ and $\mathrm{S}_{2}$ respectively, and negatively influence public tuition toward $\mathrm{P}_{\mathrm{E} 2}$.

The secondary adjustments that occur due to changing tuition levels and changing private enrollment further complicate the analysis. The adjustments continue until a new equilibrium is established.

The mathematics below identify the final effects of a change in population density after all of the repercussions have occurred and a new equilibrium is obtained. Let population density be $X_{\gamma}$. The partial derivatives are as follows.

$$
\begin{align*}
\frac{\partial S^{*}}{\partial X_{6}} & =\alpha_{6}\left[1-a\left\{\frac{(w+u)-l u k+a k b}{(a+b+c)(w+u)-l u k(a+c)-a k b}\right\}\right]+\gamma_{6}\left[\frac{-a(w+u)-a k c}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right] \\
& +\delta_{6}\left[\frac{-a(l u-a)}{(a+b+c)(w+u)-l u k(a+c)-a k b}-\frac{a}{w+u}-\frac{a k}{w+u}\left\{\frac{u(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right\}\right]  \tag{133}\\
& \frac{\partial S}{\partial X_{6}} \geq 0 \text { since } \frac{\partial S^{*}}{\partial B}<0, \frac{\partial S^{*}}{\partial W} \geq 0 \frac{\partial S^{*}}{\partial A} \geq 0 \quad \alpha_{6}>0, \gamma_{6}>0 \text { and } \delta_{6}>0
\end{align*}
$$

$$
\begin{equation*}
\frac{\partial Q_{V}^{*}}{\partial X_{6}}=\frac{-\alpha_{6} k b+\gamma_{0} k(a+c)}{(a+b+c)(w+u)-l u k(a+c)-a b k}+\delta_{6}\left[\frac{1}{w+u}+\frac{k}{w+u}\left\{\frac{u(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a k b}\right\}\right] \tag{137}
\end{equation*}
$$

$$
\frac{\partial Q_{V}}{\partial X_{6}} \frac{\geq}{<} 0 \text { since } \frac{\partial Q_{V}^{*}}{\partial A}<0 \frac{\partial Q_{V}^{*}}{\partial B}>0 \quad \frac{\partial Q_{V}^{*}}{\partial W}>0 \quad \alpha_{6}>0 \quad \gamma_{6}>0 \text { and } \delta_{6}>0
$$

## Increased Future Earnings

An increase in expected future earnings from obtaining a college education initially would positively influence the private demand curves for public and private education as well as the external benefits curve. As with the case of increased average educational attainment illustrated in Figure 19, the signs of the partial derivatives of the education variables after all adjustments and interactions have occurred are ambiguous. Let the future earnings independent variable be
represented by $X_{7}$. Then the final effects of an increase in expected future earnings are expressed by the following equations.

$$
\begin{align*}
& \frac{\partial S^{*}}{\partial X_{7}}=\alpha_{7}\left[1-a\left\{\frac{(w+u)-l u k+a k b}{(a+b+c)(w+u)-l u k(a+c)-a k b}\right\}\right]+\gamma_{7}\left[\frac{-a(w+u)-a k c}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right] \\
& +\delta_{7}\left[\frac{-a(l u-a)}{(a+b+c)(w+u)-l u k(a+c)-a k b}-\frac{a}{w+u}-\frac{a k}{w+u}\left\{\frac{u(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right\}\right]  \tag{138}\\
& \frac{\partial S^{*}}{\partial X_{7}} \geq 0 \text { since } \frac{\partial S^{*}}{\partial B}<0, \frac{\partial S^{*}}{\partial W} \geq 0 \frac{\partial S^{*}}{\partial A} \geq 0 \quad \alpha_{7}>0, \gamma_{7}>0 \text { and } \delta_{7}>0 \\
& \frac{\partial P_{E}^{*}}{\partial X_{7}}=\frac{-\alpha_{7} b(w+u)+\gamma_{7}(a+c)(w+u)+\delta_{7}[l u(a+c)+a b]}{(a+b+c)(w+u)-l u k(a+c)-a b k} \geq 0  \tag{139}\\
& \text { since } \frac{\partial P_{E}^{*}}{\partial A}<0, \frac{\partial P_{E}^{*}}{\partial B}>0 \frac{\partial P_{E}^{*}}{\partial W}>0 \alpha_{7}>0 \gamma_{7}>0 \text { and } \delta_{7}>0 \\
& \frac{\partial Q_{E}^{*}}{\partial X_{7}}=\frac{\alpha_{7}[(w+u)-l u k]+\gamma_{7}[w+u-a k]+\delta_{7}(l u-a)}{(a+b+c)(w+u)-l u k(a+c)-a b k} \frac{\geq}{<} 0  \tag{140}\\
& \text { since } \frac{\partial Q_{E}^{*}}{\partial A}>0 \frac{\partial Q_{E}^{*}}{\partial B}>0 \quad \frac{\partial Q_{E}^{*}}{\partial W} \geq 0 \quad \alpha_{7}>0, \gamma_{7}>0 \text { and } \delta_{7}>0 \\
& \frac{\partial P_{V}^{*}}{\partial X_{7}}=\frac{-\alpha_{7} u k b+\gamma_{7} u k(a+c)}{(a+b+c)(w+u)-l u k(a+c)-a b k}+\delta_{7}\left[\frac{u}{w+u}+\frac{u k}{w+u}\left\{\frac{u u(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right\}\right]  \tag{141}\\
& \frac{\partial P_{V}^{*}}{\partial X_{7}} \geq 0 \text { since } \frac{\partial P_{V}^{*}}{\partial A}<0, \frac{\partial P_{V}^{*}}{\partial B}>0, \frac{\partial P_{V}^{*}}{\partial W}>0, \quad \alpha_{7}>0 \quad \gamma_{7}>0 \quad \delta_{7}>0 \\
& \frac{\partial Q_{V}^{*}}{\partial X_{7}}=\frac{-\alpha_{7} k b+\gamma_{7} k(a+c)}{(a+b+c)(w+u)-u k(a+c)-a b k}+\delta_{7}\left[\frac{1}{w+u}+\frac{k}{w+u}\left\{\frac{u(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a k b}\right\}\right]  \tag{142}\\
& \frac{\partial Q_{V}^{*}}{\partial X_{7}}<0 \text { since } \frac{\partial Q_{\dot{V}}}{\partial A}<0 \frac{\partial Q_{V}^{*}}{\partial B}>0 \frac{\partial Q_{V}^{*}}{\partial W}>0 \quad \alpha_{7}>0 \quad \gamma_{7}>0 \text { and } \delta_{7}>0
\end{align*}
$$

## Foregone Earnings

An increase in foregone earnings initially affects both of the demand curves for higher education as well as the PVEB curve as depicted in Figure 21. An increase in foregone earnings would lead to a leftward shift in the three curves due to smaller increments to personal income and a smaller increment loss to state income. A leftward shift in the private demand curves for public and private education, $D_{E}$ and $D_{v}$, would tend to decrease public and private enrollment toward $Q_{E t}$ and $Q_{V 1}$ in Figure 21, decrease public and private tuition toward $P_{E 1}$ and $P_{V 1}$, and increase the optimal per-student subsidy toward $\mathrm{S}_{1}$.



Figure 21: The Initial Effects of an Increase in Foregone Earnings in the Higher Education Market

The initial leftward shift in PVEB to PVEB $_{1}$ would tend to decrease the optimal per-student subsidy toward $\mathrm{S}_{2}$, increase public tuition toward $\mathrm{P}_{\mathrm{E} 2}$, and decrease public enrollment toward $\mathrm{Q}_{\mathrm{EL}}$.

The ultimate effects of an increase in foregone earnings on the higher education variables are uncertain due to the secondary adjustments that occur after the initial shifts in the demand curves
and the PVEB curve. Equations (143)-(147) illustrate this uncertainty. Let foregone earnings be represented by $\mathrm{X}_{8}$. Then the final effects of an increase in foregone earnings on the higher education variables can be seen mathematically to be ambiguous. Note that $\alpha_{\mathrm{g}}<0, \gamma_{\mathrm{s}}<0$, and $\delta_{8}<0$.

$$
\begin{align*}
\frac{\partial S^{*}}{\partial X_{1}} & =\alpha_{8}\left[1-a\left\{\frac{(w+u)-l u k+a k b}{(a+b+c)(w+u)-l u k(a+c)-a k b}\right\}\right] \\
& +\gamma_{1}\left[\frac{-a(w+u)-a k c}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right] \\
& +\delta_{1}\left[\frac{-a(l u-a)}{(a+b+c)(w+u)-l u k(a+c)-a k b}-\frac{a}{w+u}-\frac{a k}{w+u}\left\{\frac{l u(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a k b}\right\}\right] \geq 0 \tag{143}
\end{align*}
$$

$$
\begin{equation*}
\frac{\partial P_{E}^{*}}{\partial X_{\mathrm{B}}}=\frac{-\alpha_{8} b(w+u)+\gamma_{\mathrm{g}}(a+c)(w+u)+\delta_{8}[L u(a+c)+a b]}{(a+b+c)(w+u)-l u k(a+c)-a b k} \geq 0 \tag{144}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial Q_{E}^{*}}{\partial X_{\mathrm{B}}}=\frac{\alpha_{\mathrm{a}}[w+u-l u k]+\gamma_{\mathrm{B}}[w+u-a k]+\delta_{\mathrm{B}}(l u-a)}{(a+b+c)(w+u)-l u k(a+c)-a b k} \geq 0 \tag{145}
\end{equation*}
$$

(146) $\frac{\partial P_{v}^{*}}{\partial X_{\mathrm{t}}}=\frac{-\alpha_{g} u k b+\gamma_{s} u k(a+c)}{(a+b+c)(w+u)-l u k(a+c)-a b k}+\delta_{s}\left[\frac{u}{w+u}+\frac{u k}{w+u}\left\{\frac{u u(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right\}\right] \geq 0$

$$
\begin{align*}
\frac{\partial Q_{V}^{*}}{\partial X_{g}} & =\frac{-\alpha_{g} k b+\gamma_{8} k(a+c)}{(a+b+c)(w+u)-l u k(a+c)-a k b}  \tag{147}\\
& +\delta_{s}\left[\frac{1}{w+u}+\frac{k}{w+u}\left\{\frac{u u(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right\}\right] \geq 0
\end{align*}
$$

## Marginal Tax Rate

An increase in the marginal tax rate would initially shift downward the private demand curves for public and private education and would shift upward the external benefits curve as seen in Figure
22. The downward shifts in the two demand curves to $\mathrm{D}_{\mathrm{E} 1}$ and $\mathrm{D}_{\mathrm{v} 1}$ respectively, tend to lower enrollments, $\mathbf{Q}_{\mathrm{E} 1}$ and $\mathbf{Q}_{\mathrm{V} 1}$, raise tuitions, $\mathbf{P}_{\mathrm{E} 1}$ and $\mathbf{P}_{\mathrm{V} 1}$, and raise the optimal per-student subsidy, $\mathbf{S}_{1}$.


Figure 22: The Initial Effects of an Increase in the Marginal Tax Rate on Higher Education Equilibrium

The initial upward shift in the external benefits curve to PVEB $_{1}$ would tend to increase the subsidy to $S_{2}$, increase enrollment to $Q_{\mathrm{E}_{2}}$, and decrease tuition to $P_{\mathrm{E} 2}$. In this case, some of the
partial derivatives of the higher education variables with respect to the ultimate effects of an increase in the marginal tax rate can be determined with the mathematics. Let the marginal tax rate be $\mathbf{X}_{\mathbf{9}}$. The final effects of an increase in the marginal tax rate are given by the following equations. Note that $\alpha_{9}>0, \gamma_{9}<0$, and $\delta_{9}<0$.

$$
\begin{align*}
\frac{\partial S^{*}}{\partial X_{9}} & =\alpha_{9}\left[1-a\left\{\frac{w+u-l u k+a k b}{(a+b+c)(w+u)-l u k(a+c)-a k b}\right\}\right]+\gamma_{9}\left[\frac{-a(w+u)-a k c}{(a+b+c)(w+u)-l u k(a+c)-a k b}\right]  \tag{148}\\
& +\delta_{9}\left[\frac{-a(l u-a)}{(a+b+c)(w+u)-l u k(a+c)-a k b}-\frac{a}{w+u}-\frac{a k}{w+u}\left\{\frac{u(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a k b}\right\}\right] \geq 0
\end{align*}
$$

$$
\begin{equation*}
\frac{\partial P_{E}^{*}}{\partial X_{9}}=\frac{-\alpha_{g} b(w+u)+\gamma_{9}(a+c)(w+u)+\delta_{g}[l u(a+c)+a b]}{(a+b+c)(w+u)-l u k(a+c)-a b k}<0 \tag{149}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial Q_{E}^{\cdot}}{\partial X_{9}}=\frac{\alpha_{9}[w+u-l u k]+\gamma_{9}[w+u-a k]+\delta_{9}(l u-a)}{(a+b+c)(w+u)-l u k(a+c)-a b k} \geq 0 \tag{150}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial P_{V}^{*}}{\partial X_{9}}=\frac{-\alpha_{9} u k b+\gamma_{9} u k(a+c)}{(a+b+c)(w+u)-l u k(a+c)-a b k}+\delta_{9}\left\{\frac{u}{w+u}+\frac{u k}{w+u}\left\{\frac{u u(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right\}\right]<0 \tag{151}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial Q_{V}^{*}}{\partial x_{9}}=\frac{-a_{9} k b+\gamma_{9} k(a+c)}{(a+b+c)(w+u)-l u k(a+c)-a k b}+\delta_{9}\left[\frac{1}{w+u}+\frac{k}{w+u}\left\{\frac{l u(a+c)+a b}{(a+b+c)(w+u)-l u k(a+c)-a k b}\right\}\right]<0 \tag{152}
\end{equation*}
$$

Equations (151) and (152) indicate that after full adjustment there is a net lowering of the private demand curve for private education. This must be so because both private tuition and enrollment decline. The ultimate decrease in public tuition is consistent with the initial decrease in
the private demand for public higher education and the increase in the external benefits curve. The uncertainty of the change in public enrollment and subsidy per student stems from the interrelatedness of the two markets for higher education.

## Migration

A decrease in migration, or an increase in the percentage of college educated students who remain in a state, would initially shift the PVEB curve upward in the public higher education market as shown in Figure 23. The rightward shift in PVEB tends to increase the optimal per-student subsidy and public enrollment toward $S_{1}$ and $Q_{E 1}$, and decrease public tuition toward $P_{E 1}$.


Figure 23: The Initial Effects of a Decrease in Migration on Higher Education Equilibrium

Let the percentage of college graduates who remain in a state be $X_{10}$. Then, equations (153) through (157) show that even after the repercussions in the private market and the feedback effects into the public market there still occurs a decrease in public tuition and an increase in public enrollment. The ultimate decrease in private tuition and private enrollment is indicative of a
leftward shift in the private demand curve for private higher education that would occur from an increase in the support of public higher education. The change in the optimal per-student subsidy for public students is unknown as seen in equation (153). Recall that $\alpha_{10}>0$.

$$
\begin{align*}
& \frac{\partial P_{E}^{*}}{\partial X_{10}}=\alpha_{10}\left[\frac{-b(w+u)}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right]<0  \tag{154}\\
& \frac{\partial Q_{E}^{*}}{\partial X_{10}}=\alpha_{10}\left[\frac{w+u-l u k}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right]>0  \tag{155}\\
& \frac{\partial P_{V}^{*}}{\partial X_{10}}=\alpha_{10}\left[\frac{-u k b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right]<0  \tag{156}\\
& \frac{\partial Q_{V}^{*}}{\partial X_{10}}=\alpha_{10}\left[\frac{-k b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right]<0
\end{align*}
$$

## Outside Funding

An increase in outside funding for public higher education institutions is expected to lower the supply curve for public higher education. Figure 24 illustrates this shift. The decrease in the public supply curve would tend to increase public enrollment and decrease the optimal per-student subsidy and public tuition.

The secondary effects of an increase in outside funding for public institutions would include a leftward shift in the private demand curve for private higher education due to decreased public tuition which would tend to decrease private tuition and enrollment. Feedback effects on the public market would occur because the PVEB curve would shift upward due to decreased private


Figure 24: The Initial Effects of an Increase in Outside Funding to Public Institutions on Higher Education Equilibrium
enrollments while the private demand curve for public education would shift left due to decreased private tuition. These adjustments continue until a new equilibrium is established.

Let outside funding be $\mathrm{X}_{11}$. Then the final effects of an increase in outside money for public institutions on the equilibrium values of the five higher education variables can be demonstrated mathematically with the following equations. Note that $\theta_{11}<0$.

$$
\begin{equation*}
\frac{\partial S^{*}}{\partial X_{11}}=\theta_{11}\left[\frac{a(w+u)-a l u k-a k b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right] \geq 0 \tag{158}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial P_{\Sigma}^{*}}{\partial X_{11}}=\theta_{11}\left[\frac{b(w+u)}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right]<0 \tag{159}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial Q_{E}^{*}}{\partial X_{11}}=\theta_{11}\left[\frac{-(w+u)+l u k}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right]>0 \tag{160}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial P_{V}^{*}}{\partial X_{11}}=\theta_{11}\left[\frac{u k b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right]<0 \tag{161}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial Q_{v}^{*}}{\partial X_{11}}-\theta_{11}\left[\frac{k b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right]<0 \tag{162}
\end{equation*}
$$

The preceding equations are consistent with a decline in the supply curve for public higher education; public enrollment would rise and public tuition would fall reflecting a movement down the public demand curve. Private tuition and enrollment would fall reflecting a leftward shift in the private demand curve for private education as students are drawn into the public market. Again, the change in the optimal per-student subsidy for public students is ambiguous.

## State Appropriations to Private Institutions

State support of private institutions in the form of appropriations and need-based subsidies to private students would lower the supply curve in the private higher education market. The rightward shift in the private supply curve would tend to increase private enrollment and decrease private tuition as seen in Figure 25.

The increase in private enrollment would tend to lower the PVEB curve and the decrease in private tuition would tend to shift the private demand curve for public higher education leftward. These changes would then have feedback effects in the private sector and the adjustments will continue until a new equilibrium is established. The final effects of an increase in state appropriations to private institutions on the higher education variables can be seen mathematically.


Figure 25: The Initial Effect of State Funding for Private Institutions on the Higher Education Market

Let state funds for private higher education institutions be represented by $\mathbf{X}_{12}$. Equations (163) through (167) illustrate the uncertainty of the final effects of a change in state funding of private schools on the five higher education variables due to feedback effects between the two markets. Recall that $\lambda_{12}<0$.

$$
\begin{align*}
\frac{\partial S^{*}}{\partial X_{12}} & =\lambda_{12}\left\{\left[\frac{-a[l w+a-a l k]}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right]+\frac{a}{w+u}\right.  \tag{163}\\
& \left.-\frac{a k}{w+u}\left[\frac{l w(a+c)-a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right]\right\} \geq
\end{align*}
$$

$$
\begin{equation*}
\frac{\partial P_{E}^{\bullet}}{\partial X_{12}}=\lambda_{12}\left[\frac{l w(a+c)-a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right] \geq 0 \tag{164}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial Q_{E}^{*}}{\partial X_{12}}=\lambda_{12}\left[\frac{(l w+a)-a l k}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right]<0 \tag{165}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial P_{v}^{*}}{\partial X_{12}}=\lambda_{12}\left[\frac{w}{w+u}+\frac{u k}{w+u}\left\{\frac{l w(a+c)-a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right\}\right] \geq 0 \tag{166}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial Q_{V}^{*}}{\partial X_{12}}=\lambda_{12}\left[-\frac{1}{w+u}+\frac{k}{w+u}\left\{\frac{l w(a+c)-a b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right\}\right] \frac{2}{<} 0 \tag{167}
\end{equation*}
$$

## Junior College Enrollment

Junior college enrollment initially affects the location of the public education supply curve. The effect of an increase in junior college enrollment is analogous to an increase in outside funding for public schools. An increase in the percentage of junior college enrollment in the public market would initially lower the public supply curve due to the cost savings associated with two-year schools as previously shown in Figure 24. Initially, this would increase total public enrollment and decrease the optimal subsidy and public tuition. The final effects of a change in junior college enrollment on the five higher education variables after all market adjustments are seen with equations (168) through (172) below. Let the percentage of junior college students in the public market be $X_{13}$. An increase in the percent of junior college enrollment lowers $S_{E}$, thus $\theta_{13}<0$.

$$
\begin{equation*}
\frac{\partial S^{*}}{\partial \mathrm{X}_{13}}=\theta_{13}\left[\frac{a(w+u)-a l u k-a k b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right] \frac{\geq}{<} 0 \tag{168}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial P_{E}^{*}}{\partial X_{13}}=\theta_{13}\left[\frac{b(w+u)}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right]<0 \tag{169}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial Q_{E}^{*}}{\partial X_{13}}=\theta_{13}\left[\frac{-(w+u)+l u k}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right]>0 \tag{170}
\end{equation*}
$$

$$
\begin{align*}
& \frac{\partial P_{V}^{*}}{\partial X_{13}}=\theta_{13}\left[\frac{u k b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right]<0  \tag{171}\\
& \frac{\partial Q_{V}^{*}}{\partial X_{13}}=\theta_{13}\left[\frac{k b}{(a+b+c)(w+u)-l u k(a+c)-a b k}\right]<0 \tag{172}
\end{align*}
$$

The decline in public tuition and the rise in public enrollment is consistent with a movement down the private demand curve for public higher education. The decrease in private tuition and enrollment indicates that a leftward shift in the private demand curve for private higher education has occurred due to the decline in public tuition. The change in the optimal per-student subsidy for public students remains ambiguous.

The preceding pages described an attempt to account for the influence of private higher education institutions on the public higher education market when changes in the identified explanatory variables occur. In the two-sector model, several predictions emerge about the directions of changes in the higher education variables, but the optimal per-student subsidy for public students is the most elusive of all the variables and it is ambiguously related to every determinant due to the link between the public and private higher education markets. Statistical techniques will be used to determine the influence of the explanatory variables on the higher education market and will provide some insight into the usefulness of the two-sector analysis versus the one-sector analysis described in a previous chapter.

## CHAPTER 7. TWO-SECTOR EMPIRICAL MODEL AND RESULTS

## Regression Procedure

The two-sector regression model includes the explanatory variables used in the one-sector model except the percentage of students enrolled in private institutions in a state. Private enrollment is not included as an explanatory variable because it is determined within the two-sector model. The five higher education variables contained in the two-sector mathematical model are included as the dependent variables. The private tuition regression includes an observation for Wyoming even though private institutions do not currently exist in that state. The private tuition observation for Wyoming included in the regression model is the average of the adjacent states' private tuition.

The two-sector regression equation can be expressed as follows.

```
SUBSTD \(-\beta_{0}+\beta_{1}\) PERPOP18 \(+\beta_{2}\) SCHOOL \(+\beta_{3}\) PERCAPIN \(+\beta_{4}\) POPLN \(+\beta_{5}\) URBAN
    \(+\beta_{6}\) POPSQMI \(+\beta_{7}\) LNFUTERN \(+\beta_{8}\) LNFOREGO \(+\beta_{9}\) AVETAXRT \(+\beta_{10}\) NETMIG
    \(+\beta_{11}\) OUTSTD \(+\beta_{12}\) PVAPSTD \(+\beta_{13}\) JUCOENRL \(+E\)
```

The regression results for the optimal per-student subsidy for the 50 states are presented in Table 4. Parameter estimates, $t$ ratios, $\mathbf{R}^{2}$, adjusted $\mathbf{R}^{2}$, and VIF statistics are stated. Tables $1 B, 2 B$, 3B, and 4B in Appendix B, provide the regression results for the other higher education variables for the $\mathbf{5 0}$ states.

## Empirical Findings

The purpose of developing the two-sector model was to take into account the interdependencies in the higher education markets and therefore have a more realistic analysis. As seen in Table 4, the two-sector model regression has a marginally higher adjusted $\mathrm{R}^{\mathbf{2}}$ and nine significant parameter estimates for the 50 states. The parameter estimates are very similar in value

Table 4. Regression Results for Two-Sector Model of Per-Student Subsidies in the $\mathbf{5 0}$ States

| Variable | Parameter Estimate | T-Values | VIF |
| :--- | ---: | ---: | ---: |
|  |  | $\mathbf{R}^{2}=0.6079$ <br> Adj. $\mathbf{R}^{2}=0.4663$ |  |
| INTERCEP | -31782 |  |  |
| PERPOP18 | 435.53 | -1.162 | 0.00 |
| SCHOOL | -345.9 | 1.152 | 1.63 |
| PERCAPIN | 0.182381 | $-1.879^{* *}$ | 5.63 |
| POPLN | 50.28 | $1.696^{* *}$ | 4.00 |
| URBAN | -39.454526 | 1.206 | 2.27 |
| POPSQMI | 1.401882 | $-3.067^{*}$ | 3.71 |
| LNFUTERN | 122.92 | 1.150 | 3.69 |
| LNFOREGO | 71.73 | $3.853^{*}$ | 2.00 |
| AVETAXRT | 485.82 | $3.907^{*}$ | 1.54 |
| NETMIG | 12.10 | $2.761^{*}$ | 1.83 |
| OUTSTD | 0.684032 | $1.844^{* *}$ | 2.30 |
| PVAPSTD | -1.453266 | $1.963^{*}$ | 2.18 |
| JUCOENRL | 13.92 | $-2.235^{*}$ | 2.27 |

[^2]and are of the same sign as in the one-sector regression. The major difference is that per capita income emerges as significant at the ten percent level.

The two-sector theory shows that the effect of a change in each of the explanatory variables on the optimal public per-student subsidy is ambiguous due to the price terms in the demand equations and the treatment of the PVEB curve. Therefore, there are no inconsistencies between the signs of the parameter estimates and the theory. The empirical results for the two-sector regression suggest the educational attainment, percent urban population, and per-student state appropriations for private institutions all negatively influence the optimal state per-student subsidy for public schools.

Per capita income, expected future earnings, foregone earnings, average tax rate, nonmigration, and outside funding all appear to positively influence the optimal state subsidy for public students.

Average educational attainment again negatively influences the optimal public per-student subsidy and positively affects public enrollment, indicating that the rightward shift in the private demand curve for public higher education outweighs the rightward shift in the external benefits curve. The parameter estimates for average educational attainment are not significant at the five or ten percent confidence level for the private tuition and private enrollment regression.

Per capita income positively influences the optimal subsidy and both public and private tuition. This seems to imply that the leftward shifts in the two supply curves hypothesized in the theory are of a greater magnitude than the rightward shifts in the two demand curves and the external benefits curve. Tuition increases in the two markets combined with an increase in the optimal subsidy can only occur with a leftward shift in the supply curves, hence, the increase in the optimal per-student subsidy in wealthier states appears to be due to increased costs rather than an increased commitment to higher education from individuals or the state.

Population is not significant in the optimal subsidy regression but emerges as significant and positive in the two enrollment regressions and the private tuition regression. The positive influence on both private tuition and enrollment indicates a rightward shift in the private demand curve for private education.

Urbanization again negatively influences the optimal public subsidy per student. The enrollment and tuition regressions shed no light on what is occurring in the two markets. It is possible that, as discussed earlier, an increase in urbanization decreases a states' ability to fund higher education due to increased demands on their budgets.

The increase in expected future earnings variable, or the rate of return from investing in higher education, positively influences the optimal subsidy and negatively affects both public and private tuition levels. This implies again that the rightward shift in PVEB outweighs the rightward
shift in the private demand for public education in the public market. The negative impact on private tuition brings up the possibility that the private demand curve for private higher education shifts left due to the increased state support of public higher education and the subsequent decline in public tuition.

The average tax rate positively affects the per-student subsidy and negatively affects the public tuition (at the .1034 level of significance). It is unknown if this is due to the rightward shift in PVEB or the leftward shift in the private demand curve for public higher education or a combination of both shifts since the enrollment regression is inconclusive. Oddly enough, an increase in the average tax rate has a positive effect on private enrollment. This is unexpected since it was predicted that the private demand curve for private higher education would shift left with an increase in the tax rate

The results for an increase in the percentage of college graduates who remain in the state, or the migration index, are similar to the previous empirical results. The optimal subsidy is positively influenced and public tuition is negatively influenced as predicted. However, public enrollment is negatively affected by the migration variable instead of positively as predicted in the theory. These results are more consistent with a decrease in the private demand curve for public education than with an increase in the external benefits curve

Foregone earnings positively influences the optimal per-student subsidy and negatively influences public tuition. This is in keeping with the previous conclusion that the leftward shift of the demand for public higher education was larger than the leftward shift of the external benefits curve.

Outside money for public institutions again positively affect the per-student subsidy and both public and private tuition. As discussed previously, this could be due to a leftward shift in the public supply curve rather than the rightward shift predicted in the theory or due to a rightward shift in the demand curve for public higher education if outside money is used by students and their families as
definitive answer. The positive influence of outside money to public institutions on private tuition is possibly due to a feedback effect from the public tuition increase since public tuition positively influences the private demand curve for private education.

State appropriations for private students negatively affects the optimal public per-student subsidy and is insignificant in the other higher education regressions. The theory was completely ambiguous with respect to all of the higher education variables except public enrollment due to the conflicting effects of the shifts in the demand curves and the external benefits curve as well as the related secondary effects.

Junior college enrollment is not significant in the subsidy regression but negatively influences public tuition. This result is consistent with the theory which suggested that junior college enrollment lowered the public supply curve. The parameter estimate is positive in the public enrollment regression but is insignificant. Junior college enrollment is also insignificant in both the private tuition and enrollment regressions.

## Regression Results for the $\mathbf{4 8}$ Contiguous States

The regression results for the $\mathbf{4 8}$ states are presented in Table 5. These results are very similar to the one-sector results. Nine of the variables are significant at the five or ten percent confidence level. Per capita income is insignificant and population and population density are significant at the ten and five percent confidence levels respectively in contrast to the 50 state, twosector results.

The results for the two-sector regressions on the other higher education variables for the 48 states are presented in Tables 5B through 8B in Appendix B.

As in the one-sector comparison of the 50 and 48 state results, some of the parameter estimates are dramatically affected by the deletion of Alaska and Hawaii. The parameter estimate for the percentage of the population college-aged changes in sign between the two models, but it is

Table 5. Regression Results for Two-Sector Model of Per-Student Subsidies in the 48 Contiguous States.

| Variable | Parameter Estimate | T-Values | $\mathbf{R}^{2}=\mathbf{0 . 5 2 8 9}$ <br> Adj. $\mathbf{R}^{2}=$ <br> $\mathbf{0 . 3 4 8 8}$ |
| :--- | ---: | ---: | ---: |
|  |  |  |  |
| INTERCEP | 10615 | 0.467 |  |
| PERPOP18 | -230.39 | -0.787 |  |
| SCHOOL | -411.4 | -3.018 |  |
| PERCAPIN | 0.09 | 1.159 |  |
| POPLN | 55.74 | $1.828^{* *}$ |  |
| URBAN | -33.07 | $-3.128^{*}$ |  |
| POPSQMI | 1.95 | $2.221^{*}$ |  |
| LNFUTERN | 69.01 | $2.398^{*}$ |  |
| LNFOREGO | 46.56 | $2.385^{*}$ |  |
| AVETAXRT | 140.95 | 1.004 |  |
| NETMIG | 13.75 | $2.719^{*}$ |  |
| OUTSTD | 0.64 | $2.479^{*}$ |  |
| PVAPSTD | -0.63 | -1.269 |  |
| JUCOENRL | 20.26 | $1.752^{* *}$ |  |

*Significant at 5 percent.
**Significant at 10 percent.
not significantly different from zero in either case. The difference in the parameter estimates again could be because Alaska and Hawaii are outliers or due to multicollinearity.

Junior college enrollment positively affects the per-student subsidy in the 48 state two-sector model just as it did in the 48 state one-sector model. Public tuition is negatively affected by junior college enrollment as predicted by the theory. However, it is uncertain if the decreases in the tuition levels support an unpredicted increase in the external benefits curve or if it supports the theory. The
levels support an unpredicted increase in the external benefits curve or if it supports the theory. The enrollment regression results do not shed any light because the parameter estimates for junior college enrollment are not significantly different from zero.

The parameter estimate for population density is positive and significant for the subsidy regression and insignificant for the other four higher education variables. As discussed in the onesector empirical results in Chapter 5, this supports the external benefits argument for funding higher education since one of the theoretical predictions was that the external benefits curve, PVEB, would shift right with an increase in population density due to the increased valuation of good neighbors.

## Regression Results for a Reduced Empirical Model

A reduced empirical model was estimated for the five higher education variables to test the sensitivity of the parameters to the deletion of several explanatory variables. The explanatory variables that were deleted from the regression equation may be sufficiently correlated with the remaining variables to warrant their exclusion. The shortened regression equation for the optimal per-student subsidy can be expressed as follows.

```
SUBSTD \(=\beta_{0}+\beta_{1}\) PERPOP18 \(+\beta_{2}\) PERCAPIN \(++\beta_{3}\) POPLN \(+\beta_{4}\) URBAN
    \(+\beta_{5}\) LNFUTERN \(+\beta_{6}\) AVETAXRT \(+\beta_{7}\) NETMIG \(+\beta_{8}\) OUTSTD \(+E\)
```

Table 6 reports regression results for the optimal per-student subsidy in the 50 states using the reduced empirical model. The regression results for the remaining four higher education variables can be found in Tables 9B through 12B in Appendix B.

As shown in Table 6, the reduced empirical model for the optimal per-student subsidy in the 50 states has an adjusted $R^{2}$ of .2801 and the values of the VIF statistics are lower than in the complete model. The empirical results are very similar to the complete empirical model results reported in Table 4. The percentage of the population college-aged, per capita income, expected

Table 6. Regression Results for Two-Sector Reduced Model of Per-Student Subsidies in the 50 States.

| Variable | Parameter Estimate | T-Values | $\begin{aligned} & R^{2}=0.3977 \\ & \text { Adj. } R^{2}=0.2801 \\ & \text { VIF } \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| INTERCEP | -10129 | -2.414 | 0.0 |
| PERPOP18 | 665.46 | 1.835** | 1.11 |
| PERCAPIN | . 16 | 1.895** | 2.05 |
| POPLN | 24.85 | 0.605 | 1.64 |
| URBAN | -25.14 | -2.040* | 2.53 |
| LNFUTERN | 97.42 | 3.102* | 1.44 |
| AVETAXRT | 317.93 | 1.759** | 1.43 |
| NETMIG | 8.31 | 1.434** | 1.33 |
| OUTSTD | . 16 | 0.514 | 1.45 |

*Significant at 5 percent.
**Significant at 10 percent.
future earnings, and the average tax rate all appear to positively influence the optimal per-student subsidy. Urbanization again appears to negatively influence the optimal per-student subsidy.

The percent of the population college-aged positively influences the optimal per-student subsidy and public tuition. The rise in tuition is consistent with a rightward shift in the private demand curve for public higher education. The increase in the optimal per-student subsidy is not consistent with the initial effects of a rightward shift in the private demand curve for public higher education and must be due to feedback effects.

Per capita income positively influences the optimal per-student subsidy and tuition in both the public and private markets for higher education. This implies that the leftward shifts in the two
supply curves outweigh the rightward shifts in the demand curves and the external benefits curve.
Population positively influences both public and private enrollment and private tuition and is insignificant in the optimal subsidy and public tuition regressions. The positive coefficients for private tuition and private enrollment indicate that a rightward shift in the private demand for private education curve has occurred.

Urbanization negatively influences the optimal per-student subsidy as before and is insignificant in the tuition and enrollment regressions.

The increase in expected future earnings variable again positively influences the optimal perstudent subsidy and negatively influences public and private tuition. The implication is that the rightward shift in PVEB outweighs the rightward shift in the private demand curve for public higher education.

The average tax rate positively influences the optimal per-student subsidy as well as private enrollment. The results from the reduced model are similar to the complete model results.

The reduced empirical model regression results for the 48 contiguous states are reported in Table 7. In this regression, only the increase in expected future earnings is significant and positively affects the optimal per-student subsidy. The regression results for the other higher education variables are reported in Tables 13B through 16B in Appendix B.

## Conclusions

The two-sector theory was developed to provide a more satisfactory theoretical analysis recognizing interdependencies between the private and public markets. The regression results are slightly better with the two-sector framework, but the two-sector model does not provide any theoretical predictions about the effects of a change in any of the explanatory variables on the optimal public per-student subsidy. The advantage of the two-sector theory is that the effects of

Table 7. Regression Results for Two-Sector Reduced Model of Per-Student Subsidies in the 48 Contiguous States

| Variable | Parameter Estimate | T-Values | $\begin{gathered} R^{2}=0.1826 \\ \text { Adj. } R^{2}=0.0149 \\ \text { VIF } \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| INTERCEP | -698.50 | -0.194 | 0.0 |
| PERPOP18 | 191.34 | 0.663 | 1.08 |
| PERCAPIN | 0.06 | 0.915 | 2.18 |
| POPLN | 35.39 | 1.117 | 1.65 |
| URBAN | -10.64 | -1.057 | 2.86 |
| LNFUTERN | 57.08 | 2.280** | 1.58 |
| AVETAXRT | 95.86 | 0.664 | 1.42 |
| NETMIG | 1.57 | 0.336 | 1.19 |
| OUTSTD | . 11 | 0.468 | 1.36 |

*Significant at 5 percent.
**Significant at 10 percent.
changes in the explanatory variables on the private market tuition and enrollment variables provide additional information about what has occurred in the higher education market.

## CHAPTER 8. CONCLUSIONS

One purpose of this dissertation was to provide a theoretical framework for analyzing interstate variation in higher education expenditures. Previous studies of state government expenditures on higher education were primarily ad hoc in nature. Two theoretical models were developed to generate predictions about the influence of the explanatory variables on the optimal per-student subsidy for public higher education. In the empirical analysis, many of the theoretical predictions were confirmed, but some characteristics of the data are not explained by the theory presented here.

The most significant empirical finding of this study is that state governments appear to respond to the equity interest they have in the education of their citizens. The variables that heavily affect the return that states earn from higher education investment are consistently significant in the empirical analysis. Increased future earnings, foregone earnings, tax rate and migration influence future state government income and thus affect state support of public higher education.

The empirical analysis also confirms that state support of private institutions decreases allocations to public institutions. This result is consistent with the one-sector prediction and indicates that public institutions compete with private institutions not only for students but for state funding as well. Thus, state support of private colleges and universities is at least partially offset by decreased subsidies to public colleges and universities.

## REFERENCES

Advisory Commission on Intergovernmental Relations. Significant Features of Fiscal Federalism, Vol. 1, Washington, D.C., 1988.

Bahl, Roy W. and Robert J. Saunders. "Determinants of Changes in State and Local Government Expenditure." National Tax Journal 18, no. 1 (March, 1965): 50-57.

Becker, Gary S. Human Capital. New York:National Bureau of Economic Research, 1964.
Borus, Michael E. and Susan A. Carpenter. "Factors Associated with College Attendance of High School Seniors." Economics of Education_Review 3, no. 3 (1984):169-176.

Bowen, Howard R. The Costs of Higher Education. San Francisco: Jossey-Bass Publishers, 1987.
Bowles, Samuel. "Schooling and Inequality from Generation to Generation." Journal of Political Economy 80, no. 3 (1972):S219-S251.

Campbell, Robert and Barry N. Siegel. "The Demand for Higher Education in the United States, 1919-1964." American Economic Review 57 (June 1967): 482-494.

Cohn, Elchanan. The Economics of Higher Education. Cambridge, MA.: Ballinger Publishers, 1979.

Cohn, Elchanan. "Factors Affecting Variation in Faculties Salaries and Compensation in Higher Education." Journal of Higher Education 44 (1973):124-136.

Cohn, Elchanan and J. Morgan. "The Demand for Higher Education." Higher Educational Review 1 (1978):18-30.

Corrazini, A. J., D. J. Dugan, and H. G. Grabowski. "Determinants and Distributional Aspects of Enrollment in U.S. Higher Education." Journal of Human Resources 7 (1972):39-50.

Coughlin, Cletus C. and O. Homer Erekson. "Determinants of State Aid and Voluntary Support of Higher Education." Economics of Education Review 5, no. 2 (1986):179-190.

Fabricant, Solomon. The Trend of Government Activity in the United States Since 1900. New York: National Bureau of Economic Research, 1952.

Fisher, Glenn. "Determinants of State and Local Government Expenditures." National Tax Journal 14, no. 4 (December 1961): 349-354.

Fisher, Glenn. "Interstate Variation in State and Local Government Expenditure." National Tax Journal 17, no. 1 (March 1964): 57-74.

Freeman, Richard B. "Demand for Education." In Handbook of Labor Economics, ed. 0. Ashenfelter and R. Layard, 359-386. New York, NY, Elsevier Science Publishers, 1986.

Galper, H. and R. Dunn, Jr. "A Short-Run Demand Function for Higher Education in the United States." Journal of Political Economy (September 1969): 765-777.

Gold, Steven D. "State Support of Higher Education: A National Perspective." Planning for Higher Education 18, no. 3 (1989): 21-33.

Hansen, W. L. and B. A. Weisbrod. Benefits, Costs, and Finance of Public. Hipher Education. Chicago: Markham, 1969.

Hoenack, Stephen and Eileen Collins (ed.). The Economics of American Universities: Management, Operations and Fiscal Environment, Albany, NY: State University of New York Press, 1990.

Holcombe, Randall G. and Lora P. Holcombe. "The Return to the Federal Government from Investment in Higher Education." Public Finance Ouarterly 12, no. 3 (1984): 365-371.

Jackson, Gregory A. and George B. Weathersby. "Individual Demand for Higher Education." Journal of Higher Education 46 (September 1975): 623-652.

Jaschik, Scott. "State Funds for Higher Education Drop in Year; First Decline Since Survey Began 33 Years Ago." The Chronicle of Higher Education, 6 November 1991, 1, A38.

Judge, George G., R. Carter Hill, William E. Griffiths, Helmut Lutkepohl, Tsoung-Chao Lee. Introduction to the Theory and Practice of Econometrics. 2nd ed. New York: Wiley, 1988.

Lazear, E. "Education: Consumption or Production?" Journal of Political Economy 85, no. 3. (1977):569-598.

Lehr, Dona K. and Jan M. Newton. "Time Series and Cross-Sectional Investigations of the Demand for Higher Education." Economic Inquiry 16 (July 1978): 411-422.

Leslie, Larry L. "The Financial Prospects for Higher Education in the 1980s." Journal of Higher Education 51, no. 1 (1980):1-17.

Leslie, Larry L. and Garey Ramsey. "State Appropriations and Enrollments: Does Enrollment Growth Still Pay?" Journal of Higher Education 57, no. 1 (January 1986): 1-19.

Leslie, Larry L. and Paul T. Brinkman. "Student Price Response in Higher Education: The Student Demand Studies." Journal of Higher Education 58, no. 2 (March, 1987): 181-203.

Leslie, Larry L and Paul T. Brinkman. The Economics of Higher Education. New York: McMillan Publishing Company, 1988.

Maddala, G. S. Introduction to Econometrics. 1st ed. New York: McMillan Publishing Company, 1988.

Manski, C. F. and D. A. Wise. College Choice in America. Cambridge, MA: Harvard University Press, 1983.

Mattila, J. Peter. "Determinants of Male School Enrollments: A Time Series Analysis." Review of Economics and Statistics 64 (1982): 242-251.

McIntyre, M. Charles. "Determinants of Expenditures for Public Higher Education." National Tax Journal 22, no. 2 (June, 1969): 262-272.

McMahon, W. W. Investment in Higher Education. Lexington: D C Heath and Company, 1974.
McMahon, W.W. "Consumption and Other Benefits of Education." In Economics of Education: Research and Studies, ed. George Psacharopoulos, 129-132. New York: Pergamon Press, 1987a.

McMahon, W.W. "Externalities in Education." In Economics of Education: Research and Studies, ed. George Psacharopoulos, 133-140. New York: Pergamon Press, 1987 b .

McMahon, W.W. "Expected Rates of Returns to Education." In Economics of Education; Research and Studies, ed. George Psacharopoulos, 133-140. New York: Pergamon Press, 1987c.

Mincer, Jacob. Schooling, Experience and Earnings. New York: Columbia University Press, 1974.
Musgrave, Richard A. and Peggy B. Musgrave. Public. Finance in Theory and Practice. 5th ed. New York, McGraw Hill, Inc., 1989.

Parsons, Donald O. "The Cost of School Time, Foregone Earnings, and Human Capital Formation." Journal of Political Economy 82 (1974):251-266.

Peltzman, Sam. "The Effect of Government Subsidies-in-Kind on Private Expenditures: The Case of Higher Education." Journal of Political Economy 81, No. 1 (1973):1-27.

Peterson, Robert G. "Environmental and Political Determinants of State Higher Education Appropriations Policies." Journal of Higher Education 47, No. 4 (1976):523-542.

Radner, R. and L.S. Miller. "Demand and Supply in U.S. Higher Education: A Progress Report." American Economic Review 60 (1970):326-3.

Sacks, Seymour and Robert Harris. "The Determinants of State and Local Government Expenditures and Intergovernmental Flows of Funds." National Tax Journal 17, No. 1 (March 1964): 75-85.
U.S. Bureau of the Census. Statistical Abstract of the United States, 1990. 110 ed. Washington, D.C.: U.S. Government Printing Office, 1990.
U.S. Bureau of the Census. 1980 Census of Population. Washington, D.C.: U.S. Government Printing Office, 1980.
U.S. Bureau of Economic Analysis. Survey of Current Business 71, No. 12, December, U.S. Government Printing Office, 1991.
U.S. Department of Education. Integrated Postsecondary Education Data System; Enrollment 1988-89. [data tape] U.S. Department of Education, Washington, D.C.
U.S. Department of Education. Integrated Postsecondary Education Data System: Finance FY 1988-89. [data tape] U.S. Department of Education, Washington, D.C.
U.S. Department of Education. State Higher Education Profiles. Washington, D.C.: U.S. Government Printing Office, 1990.

Verry, D.W. "Educational Cost Functions." In Economics of Education: Research and Studies, ed. George Psacharopoulos, 400-408. New York: Pergamon Press, 1987.

Willis, Robert J. In Handbook of Labor Economics, Vol. 1, ed. O. Ashenfelter and R. Layard. Elsevier Science Publishers, New York, NY, 1986.

Windham, D.M. "Social Benefits and the Subsidization of Higher Education: A Critique." Higher Education 5 (1976):237-252.

Wiseman, J. "Public Finance in Education." In Economics of Education; Research and Studies, ed. George Psacharopoulos, 436-438. New York: Pergamon Press, 1987.

Woodhall, M. "Cost Analysis in Education." In Economics of Education; Research and Studies, ed. George Psacharopoulos, 393-399. New York: Pergamon Press, 1987.

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## APPENDIX A

Table 1A. Regression Results for One-Sector Model of Public Tuition in the $\mathbf{5 0}$ States.

| Variable | Parameter Estimate | T-Values | $\begin{gathered} R^{2}=0.6276 \\ \text { Adj. } R^{2}=0.4787 \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| INTERCEP | 28728 | 1.557 |  |
| PERPOP18 | 202.47 | 0.859 |  |
| SCHOOL | -0.54 | -0.005 |  |
| PERCAPIN | 0.13 | 1.991* |  |
| POPLN | 14.45 | 0.542 |  |
| URBAN | 7.96 | 0.983 |  |
| POPSQMI | -0.60 | -0.779 |  |
| LNFUTERN | -59.52 | -2.718* |  |
| LNFOREGO | -31.56 | -2.412* |  |
| AVETAXRT | -180.92 | -1.652** |  |
| NETMIG | -7.90 | -1.936** |  |
| OUTSTD | 0.53 | 2.465* |  |
| PVORIENT | 5.31 | 0.505 |  |
| PVAPSTD | 0.20 | 0.496 |  |
| JUCOENRL | -30.38 | -3.062* |  |

[^3]Table 2A. Regression Results for One-Sector Model of Public Enrollment for the 50 States.

| Variable | Parameter Estimate | T-Values | $\mathbf{R}^{2}=0.9749$ <br> Adj. $R^{2}=0.9649$ |
| :--- | :---: | :---: | :---: |
| INTERCEP | -1646711 | -1.919 |  |$|$

*Significant at 5 percent.
**Significant at 10 percent.

Table 3A. Regression Results for One-Sector Model of Public Tuition in the 48 Contiguous States.

| Variable | Parameter Estimate | T-Values | $\mathbf{R}^{2}=0.6638$ <br> Adj. $R^{2}=0.5212$ |
| :--- | :---: | :---: | :---: | :---: |
| INTERCEP |  |  |  |
| PERPOP18 | 29653 | 1.455 |  |

[^4]Table 4A. Regression Results for One-Sector Model of Public Enrollment for the $\mathbf{4 8}$ Contiguous States.

| Variable | Parameter Estimate | T-Values | $\begin{gathered} \mathbf{R}^{2}=0.9770 \\ \text { Adj. } \mathbf{R}^{2}=0.9673 \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| INTERCEP | -1714121 | -1.804 |  |
| PERPOP18 | 43803.77 | 3.790* |  |
| SCHOOL | 16221.6 | 3.018* |  |
| PERCAPIN | -2.90 | -0.882 |  |
| POPLN | 30497.67 | 24.856* |  |
| URBAN | 66.74 | 0.158 |  |
| POPSQMI | -8.99 | -0.256 |  |
| LNFUTERN | -884.45 | -0.730 |  |
| LNFOREGO | -693.67 | -0.835 |  |
| AVETAXRT | -755.54 | -0.137 |  |
| NETMIG | -440.94 | -2.214* |  |
| OUTSTD | -14.79 | -1.437 |  |
| PVORIENT | -1353.61 | -2.824* |  |
| PVAPSTD | -14.33 | -0.730 |  |
| JUCOENRL | 264.72 | 0.565 |  |

[^5]
## APPENDIX B

Table 1B. Regression Results for Two-Sector Model of Public Tuition for the 50 States.

| Variable | Parameter Estimate | T-Values | $R^{2}=0.6249$ <br> Adj. $\mathbf{R}^{\mathbf{2}}=\mathbf{0 . 4 8 9 4}$ |
| :---: | :---: | :---: | :---: |
| INTERCEP | 32322 | 1.918 |  |
| PERPOP18 | 209.11 | 0.898 |  |
| SCHOOL | -6.22 | -0.055 |  |
| PERCAPIN | 0.14 | 2.180* |  |
| POPLN | 17.57 | 0.685 |  |
| URBAN | 8.61 | 1.087 |  |
| POPSQMI | -0.52 | -0.706 |  |
| LNFUTERN | -64.20 | -3.268* |  |
| LNFOREGO | -34.79 | -3.078* |  |
| AVETAXRT | -181.04 | -1.671 |  |
| NETMIG | -7.90 | -1.955** |  |
| OUTSTD | 0.53 | 2.500* |  |
| PVAPSTD | 0.18 | 0.463 |  |
| JUCOENRL | -31.71 | -3.346* |  |

[^6]Table 2B. Regression Results for Two-Sector Model of Public Enrollment for the $\mathbf{5 0}$ States.

| Variable | Parameter Estimate | T-Values | $\mathbf{R}^{2}=0.9703$ <br> Adj. $\mathbf{R}^{2}=0.9596$ |
| :--- | :---: | :---: | :---: | :---: |
| INTERCEP | -2482450 | -2.925 |  |$|$

[^7]Table 3B. Regression Results for the Two-Sector Model of Private Tuition for the 50 States.

| Variable | Parameter Estimate | T-Values | $\begin{aligned} \mathbf{R}^{2} & =0.5852 \\ \text { Adj. } \mathbf{R}^{\mathbf{2}} & =0.4354 \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| INTERCEP | 81775 | 2.051 |  |
| PERPOP18 | -262.46 | -. 476 |  |
| SCHOOL | -328.1 | -1.223 |  |
| PERCAPIN | 0.34 | 2.187* |  |
| POPLN | 132.62 | 2.183* |  |
| URBAN | 16.21 | . 865 |  |
| POPSQMI | . 95 | . 539 |  |
| LNFUTERN | -137.47 | -2.957* |  |
| LNFOREGO | -41.18 | -1.540 |  |
| AVETAXRT | -75.35 | -. 294 |  |
| NETMIG | 3.85 | . 403 |  |
| OUTSTD | . 98 | 1.934** |  |
| PVAPSTD | -. 19 | -. 204 |  |
| JUCOENRL | -14.77 | -. 659 |  |

[^8]Table 4B. Regression Results for the Two-Sector Model of Private Enrollment for the 50 States.

| Variable | Parameter Estimate | T-Values | $\begin{aligned} \mathbf{R}^{2} & =0.7113 \\ \text { Adj. } \mathbf{R}^{2} & =0.6071 \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| INTERCEP | 949350 | 0.937 |  |
| PERPOP18 | -436.12 | -0.031 |  |
| SCHOOL | -3839.4 | -0.563 |  |
| PERCAPIN | 4.17 | 1.048 |  |
| POPLN | 8309.86 | 5.382* |  |
| URBAN | 233.05 | 0.489 |  |
| POPSQMI | 2.23 | 0.049 |  |
| LNFUTERN | -1263.45 | -1.069 |  |
| LNFOREGO | -666.15 | -0.980 |  |
| AVETAXRT | 13419.93 | 2.060* |  |
| NETMIG | -87.15 | -0.359 |  |
| OUTSTD | -5.53 | -0.429 |  |
| PVAPSTD | 0.93 | 0.039 |  |
| JUCOENRL | -503.16 | -0.883 |  |

[^9]Table 5B. Regression Results for the Two-Sector Model of Public Tuition for the $\mathbf{4 8}$ Contiguous States.

| Variable | Parameter Estimate | T-Values | $\begin{gathered} R^{2}=0.6631 \\ \text { Adj. } R^{2}=0.5343 \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| INTERCEP | 31458 | 1.664 |  |
| PERPOP18 | 371.96 | 1.527 |  |
| SCHOOL | 42.54 | 0.375 |  |
| PERCAPIN | 0.14 | 2.081* |  |
| POPLN | 10.08 | 0.397 |  |
| URBAN | 12.24 | 1.392 |  |
| POPSQMI | -0.74 | -1.015 |  |
| LNFUTERN | -66.64 | -2.783* |  |
| LNFOREGO | -42.83 | -2.636* |  |
| AVETAXRT | -122.23 | -1.047 |  |
| NETMIG | -10.17 | -2.417* |  |
| OUTSTD | 0.46 | 2.134* |  |
| PVAPSTD | 0.09 | 0.241 |  |
| JUCOENRL | -29.81 | -3.098* |  |

[^10]Table 6B. Regression Results for Two-Sector Model of Public Enrollment for the 48 Contiguous States.

| Variable | Parameter Estimate | T-Values | $\mathbf{R}^{2}=0.9715$ <br> Adj. $\mathbf{R}^{2}=0.9605$ |
| :--- | :---: | :---: | :---: |
| INTERCEP | -2627332 | -2.679 |  |$|$

[^11]Table 7B. Regression Results for the Two-Sector Model of Private Tuition for the 48 Contiguous States.

| Variable | Parameter Estimate | T-Values | $\mathbf{R}^{2}=0.6372$ <br> Adj. $\mathbf{R}^{2}=0.4985$ |
| :--- | :---: | :---: | :---: | :---: |
| INTERCEP | 67934 | 1.570 |  |

[^12]Table 8B. Regression Results for the Two-Sector Model of Private Enrollment for the 48 Contiguous States.

| Variable | Parameter Estimate | T-Values | $\begin{gathered} R^{2}=0.7140 \\ \text { Adj. } R^{2}=0.6047 \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| INTERCEP | 798487 | 0.670 |  |
| PERPOP18 | 4826.62 | 0.314 |  |
| SCHOOL | -2687.0 | -0.375 |  |
| PERCAPIN | 4.39 | 1.025 |  |
| POPLN | 8147.39 | 5.091* |  |
| URBAN | 283.23 | 0.510 |  |
| POPSQMI | -3.70 | -0.080 |  |
| LNFUTERN | -1140.25 | -0.755 |  |
| LNFOREGO | -742.36 | -0.724 |  |
| AVETAXRT | 15650.58 | 2.125* |  |
| NETMIG | -136.44 | -0.514 |  |
| OUTSTD | -6.81 | -0.496 |  |
| PVAPSTD | -3.32 | -0.127 |  |
| JUCOENRL | -486.48 | -0.801 |  |

*Significant at 5 percent.
**Significant at 10 percent.

Table 9B. Regression Results for the Reduced Two-Sector Model of Public Tuition for the $\mathbf{5 0}$ States.

| Variable | Parameter Estimate | T-Values | $\begin{gathered} \mathbf{R}^{2}=0.3890 \\ \text { Adj. } \mathbf{R}^{2}=0.2698 \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| INTERCEP | -2925.00 | -1.099 |  |
| PERPOP18 | 503.81 | 2.190** |  |
| PERCAPIN | 0.12 | 2.281** |  |
| POPLN | 5.73 | 0.220 |  |
| URBAN | -6.17 | -0.790 |  |
| LNFUTERN | -46.80 | $-2.350^{* *}$ |  |
| AVETAXRT | -174.27 | -1.520 |  |
| NETMIG | -6.09 | -1.657 |  |
| OUTSTD | 0.43 | 2.065 |  |

*Significant at 5 percent.
**Significant at 10 percent.

Table 10B. Regression Results for the Reduced Two-Sector Model of Public Enrollment for the 50 States.

| Variable | Parameter Estimate | T-Values | $\begin{gathered} \mathbf{R}^{2}=0.95 \\ \text { Adj. } R^{2}=0.94 \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| INTERCEP | 4075.57 | 0.031 |  |
| PERPOP18 | 6679.46 | 0.579 |  |
| PERCAPIN | -3.10 | -1.091 |  |
| POPLN | 30.16 | 23.067** |  |
| URBAN | -30.58 | -0.078 |  |
| AVETAXRT | -2558.99 | -0.445 |  |
| NETMIG | -7.47 | -0.041 |  |
| OUTSTD | 4.64 | 0.443 |  |

*Significant at 5 percent.
**Significant at 10 percent.

Table 11B. Regression Results for the Reduced Two-Sector Model of Private Tuition for the 50 States.

| Variable | Parameter Estimate | T-Values | $\mathbf{R}^{2}=0.4720$ <br> Adj. $\mathbf{R}^{\mathbf{2}}=\mathbf{0 . 3 6 9 0}$ |
| :---: | :---: | :---: | :---: |
| INTERCEP | -2132.24 | -0.383 |  |
| PERPOP18 | 457.35 | 0.951 |  |
| PERCAPIN | 0.31 | 2.650** |  |
| POPLN | 106.69 | 1.957* |  |
| URBAN | 10.12 | 0.619 |  |
| LNFUTERN | -96.73 | -2.322** |  |
| AVETAXRT | -92.63 | -0.386 |  |
| NETMIG | -3.05 | -0.397 |  |
| OUTSTD | 0.47 | 1.091 |  |

*Significant at 5 percent.
**Significant at 10 percent.

Table 12B. Regression Results for the Reduced Two-Sector Model of Private Enrollment for the 50 States.

| Variable | Parameter Estimate | T-Values | $R^{2}=0.6836$ |
| :--- | :---: | :---: | :---: |
|  |  |  |  |

[^13]Table 13B. Regression Results for the Reduced Two-Sector Model of Public Tuition for the 48 Contiguous States.

| Variable | Parameter Estimate | T-Values | $\mathbf{R}^{2}=0.4426$ |
| :--- | :---: | :---: | :---: | :---: |
| Adj. $\mathbf{R}^{2}=0.3283$ |  |  |  |

[^14]Table 14B. Regression Results for the Reduced Two-Sector Model of Public Enrollment for the 48 Contiguous States.

| Variable | Parameter Estimate | T-Values | $\begin{aligned} \mathbf{R}^{2} & =0.9510 \\ \text { Adj. } R^{2} & =0.9409 \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| INTERCEP | 29315 | 0.190 |  |
| PERPOP18 | 5455.76 | 0.440 |  |
| PERCAPIN | -3.42 | -1.119 |  |
| POPLN | 30,182.05 | 22.77** |  |
| URBAN | 17.20 | 0.040 |  |
| LNFUTERN | -959.03 | -0.892 |  |
| AVETAXRT | -3101.09 | -0.501 |  |
| NETMIG | -29.21 | -0.145 |  |
| OUTSTD | 4.45 | 0.413 |  |

[^15]Table 15B. Regression Results for the Reduced Two-Sector Model of Private Tuition for the 48 ContIguous States.

| Variable | Parameter Estimate | T-Values | $\begin{gathered} R^{2}=0.5653 \\ \text { Adj. } R^{2}=0.4761 \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| INTERCEP | -10295 | -1.809 |  |
| PERPOP18 | 906.10 | 1.984* |  |
| PERCAPIN | 0.36 | 3.266** |  |
| POPLN | 84.54 | 1.686* |  |
| URBAN | 5.22 | 0.328 |  |
| LNFUTERN | -71.39 | -1.802* |  |
| AVETAXRT | 143.95 | 0.630 |  |
| NETMIG | -0.36 | -0.049 |  |
| OUTSTD | 0.47 | 1.198 |  |

[^16]Table 16B. Regression Results for the Reduced Two-Sector Model of Private Enrollment for the 48 Contiguous States.

| Variable | Parameter Estimate | T-Values | $\begin{gathered} \mathbf{R}^{2}=0.6938 \\ \text { Adj. } \mathbf{R}^{2}=0.6310 \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| INTERCEP | -315763 | -2.130 |  |
| PERPOP18 | 15342.74 | 1.289 |  |
| PERCAPIN | 4.29 | 1.462 |  |
| POPLN | 775.55 | 5.935* |  |
| URBAN | -53.63 | -0.129 |  |
| LNFUTERN | -405.86 | -0.393 |  |
| AVETAXRT | 15826.64 | $2.660^{*}$ |  |
| NETMIG | -111.38 | -0.576 |  |
| OUTSTD | -11.76 | -1.137 |  |

[^17]
## APPENDIX C

Summary Statistics

| Variable | Mean | Std Dev | Minimum | Maximum |
| :---: | :---: | :---: | :---: | :---: |
| STUDSUB | 4751.88 | 1404.12 | 2864.79 | 11682.10 |
| PUBPRICE | 1903.41 | 884.11 | 953.13 | 5864.43 |
| PUBENROL | 141875.04 | 158347.03 | 13046.00 | 946323.00 |
| PVTPRICE | 6553.64 | 1989.21 | 2113.11 | 10713.88 |
| PVTENROL | 43396.48 | 60605.56 | 0 | 325512.0 |
| PERPOP18 | . 109 | . 004 | . 096 | . 118 |
| SCHOOL | 12.48 | . 18 | 12.10 | 12.8 |
| PERCAPIN | 15571.90 | 2727.76 | 11116.00 | 23059.00 |
| POPLN | 4911.00 | 5307.09 | 479.00 | 28314.00 |
| URBAN | 63.80 | 21.96 | 20.00 | 100.00 |
| POPSQMI | 164.04 | 231.19 | 1.00 | 1034.00 |
| LNFUTERN | . 241 | . 065 | . 105 | . 352 |
| LNFOREGO | 8.92 | . 099 | 8.73 | 9.20 |
| AVETAXRT | 0.088 | . 011 | . 067 | . 125 |
| NETMIG | 1.06 | . 33 | . 56 | 2.21 |
| OUTSTD | 2326.61 | 621.68 | 1424.92 | 4181.26 |
| PVORIENT | . 210 | . 132 | 0 | . 600 |
| PVAPSTD | 313.07 | 339.70 | 0 | 1184.26 |
| JUCOENRL | . 306 | . 134 | 0 | . 577 |


[^0]:    * Significant at 5 percent.
    **Significant at 10 percent.

[^1]:    * Significant $\alpha=.05$.
    **Significant $\alpha=.10$.

[^2]:    *Significant at 5 percent.
    **Significant at 10 percent.

[^3]:    *Significant at 5 percent.
    **Significant at 10 percent.

[^4]:    *Significant at 5 percent.
    **Significant at 10 percent.

[^5]:    *Significant at 5 percent.
    **Significant at 10 percent.

[^6]:    *Significant at 5 percent.
    **Significant at 10 percent.

[^7]:    *Significant at 5 percent.
    **Significant at 10 percent.

[^8]:    *Significant at 5 percent.
    **Significant at 10 percent.

[^9]:    *Significant at 5 percent.
    **Significant at 10 percent.

[^10]:    *Significant at 5 percent.
    **Significant at 10 percent.

[^11]:    *Significant at 5 percent.
    **Significant at 10 percent.

[^12]:    *Significant at 5 percent.
    **Significant at 10 percent.

[^13]:    *Significant at 5 percent.
    **Significant at 10 percent.

[^14]:    *Significant at 5 percent.
    **Significant at 10 percent.

[^15]:    *Significant at 5 percent.
    **Significant at 10 percent.

[^16]:    *Significant at 5 percent.
    **Significant at 10 percent.

[^17]:    *Significant at 5 percent.
    **Significant at 10 percent.

